ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY, PALKULAM

Department of Electrical and Electronics Engineering

Training and Placement Certification Course

TRP 1301

APTITUDE SKILL – PHASE I

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1. NUMBERS

IMPORTANT FACTS AND FORMULAE

I..Numeral : In Hindu Arabic system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called ***digits*** to represent any number.

A group of digits, denoting a number is called a ***numeral.***

We represent a number, say 689745132 as shown below :

Ten Crores (10^8)	Crores (10^7)	Ten Lacs (Millions) (10^6)	Lacs (10^5)	Ten Thousands (10^4)	Thousands (10^3)	Hundreds (10^2)	Tens (10^1)	Units (10^0)
6	**8**	**9**	**7**	**4**	**5**	**1**	**3**	**2**

We read it as : 'Sixty-eight crores, ninety-seven lacs, forty-five thousand, one hundred and thirty-two'.

II Place Value or Local Value of a Digit in a Numeral :

In the above numeral :

Place value of 2 is (2 x 1) = 2; Place value of 3 is (3 x 10) = 30;

Place value of 1 is (1 x 100) = 100 and so on.

Place value of 6 is $6 \times 10^8 = 600000000$

III.Face Value : The ***face value*** of a digit in a numeral is the value of the digit itself at whatever place it may be. In the above numeral, the face value of 2 is 2; the face value of 3 is 3 and so on.

IV.TYPES OF NUMBERS

1.Natural Numbers : Counting numbers 1, 2, 3, 4, 5,..... are called ***natural*** *numbers.*

2.Whole Numbers : All counting numbers together with zero form the set of ***whole numbers***. Thus,

(i) 0 is the only whole number which is not a natural number.

(ii) Every natural number is a whole number.

3.Integers : All natural numbers, 0 and negatives of counting numbers *i.e.,* {…, -3, -2, -1, 0, 1, 2, 3,…..} together form the set of integers.

(i) **Positive Integers** : {1, 2, 3, 4, …..} is the set of all positive integers.

(ii) **Negative Integers** : {- 1, - 2, - 3,…..} is the set of all negative integers.

(iii) **Non-Positive and Non-Negative Integers** : 0 is neither positive nor negative. So, {0, 1, 2, 3,….} represents the set of non-negative integers, while {0, -1, -2, -3, …..} represents the set of non-positive integers.

4. Even Numbers : A number divisible by 2 is called an even number, e.g., 2, 4, 6, 8, 10, etc.

5. Odd Numbers : A number not divisible by 2 is called an odd number. e.g., 1, 3, 5, 7, 9, 11, etc.

6. Prime Numbers : A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

Prime numbers upto 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Prime numbers Greater than 100 : Let p be a given number greater than 100. To find out whether it is prime or not, we use the following method :

Find a whole number nearly greater than the square root of p. Let k > *jp. Test whether p is divisible by any prime number less than k. If yes, then p is not prime. Otherwise, p is prime.

e.g,,We have to find whether 191 is a prime number or not. Now, 14 > V191.

Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.

191 is not divisible by any of them. So, 191 is a prime number.

7.Composite Numbers : Numbers greater than 1 which are not prime, are known as composite numbers, e.g., 4, 6, 8, 9, 10, 12.

Note : (i) 1 is neither prime nor composite.

(ii) 2 is the only even number which is prime.

(iii) There are 25 prime numbers between 1 and 100.

8. Co-primes : Two numbers a and b are said to be co-primes, if their H.C.F. is 1. e.g., (2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes,

V.TESTS OF DIVISIBILITY

1. Divisibility By 2 : A number is divisible by 2, if its unit's digit is any of 0, 2, 4, 6, 8.

Ex. 84932 is divisible by 2, while 65935 is not.

2. Divisibility By 3 : A number is divisible by 3, if the sum of its digits is divisible by 3.

Ex.592482 is divisible by 3, since sum of its digits = (5 + 9 + 2 + 4 + 8 + 2) = 30, which is divisible by 3.

But, 864329 is not divisible by 3, since sum of its digits =(8 + 6 + 4 + 3 + 2 + 9) = 32, which is not divisible by 3.

3. Divisibility By 4 : A number is divisible by 4, if the number formed by the last two digits is divisible by 4.

Ex. 892648 is divisible by 4, since the number formed by the last two digits is 48, which is divisible by 4.

But, 749282 is not divisible by 4, since the number formed by the last tv/o digits is 82, which is not divisible by 4.

4. Divisibility By 5 : A number is divisible by 5, if its unit's digit is either 0 or 5. Thus, 20820 and 50345 are divisible by 5, while 30934 and 40946 are not.

5. Divisibility By 6 : A number is divisible by 6, if it is divisible by both 2 and 3. Ex. The number 35256 is clearly divisible by 2.

Sum of its digits = (3 + 5 + 2 + 5 + 6) = 21, which is divisible by 3. Thus, 35256 is divisible by 2 as well as 3. Hence, 35256 is divisible by 6.

6. Divisibility By 8 : A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.

Ex. 953360 is divisible by 8, since the number formed by last three digits is 360, which is divisible by 8.

But, 529418 is not divisible by 8, since the number formed by last three digits is 418, which is not divisible by 8.

7. Divisibility By 9 : A number is divisible by 9, if the sum of its digits is divisible by 9.

Ex. 60732 is divisible by 9, since sum of digits * (6 + 0 + 7 + 3 + 2) = 18, which is divisible by 9.

But, 68956 is not divisible by 9, since sum of digits = (6 + 8 + 9 + 5 + 6) = 34, which is not divisible by 9.

8. Divisibility By 10 : A number is divisible by 10, if it ends with 0.

Ex. 96410, 10480 are divisible by 10, while 96375 is not.

9. Divisibility By 11 : A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.

Ex. The number 4832718 is divisible by 11, since :

(sum of digits at odd places) - (sum of digits at even places)

(8 + 7 + 3 + 4) - (1 + 2 + 8) = 11, which is divisible by 11.

10. Divisibility By 12 ; A number is divisible by 12, if it is divisible by both 4 and 3.

Ex. Consider the number 34632.

(i) The number formed by last two digits is 32, which is divisible by 4,

(ii) Sum of digits = (3 + 4 + 6 + 3 + 2) = 18, which is divisible by 3. Thus, 34632 is divisible by 4 as well as 3. Hence, 34632 is divisible by 12.

11. Divisibility By 14 : A number is divisible by 14, if it is divisible by 2 as well as 7.

12. Divisibility By 15 : A number is divisible by 15, if it is divisible by both 3 and 5.

13. Divisibility By 16 : A number is divisible by 16, if the number formed by the last4 digits is divisible by 16.

Ex.7957536 is divisible by 16, since the number formed by the last four digits is 7536, which is divisible by 16.

14. Divisibility By 24 : A given number is divisible by 24, if it is divisible by both3 and 8.

15. Divisibility By 40 : A given number is divisible by 40, if it is divisible by both 5 and 8.

16. Divisibility By 80 : A given number is divisible by 80, if it is divisible by both 5 and 16.

Note : If a number is divisible by p as well as q, where p and q are co-primes, then the given number is divisible by pq.

If p arid q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

Ex. 36 is divisible by both 4 and 6, but it is not divisible by (4x6) = 24, since 4 and 6 are not co-primes.

VI MULTIPLICATION BY SHORT CUT METHODS

1. Multiplication By Distributive Law :

(i) a x (b + c) = a x b + a x c (ii) ax(b-c) = a x b-a x c.

Ex. (i) 567958 x 99999 = 567958 x (100000 - 1)

= 567958 x 100000 - 567958 x 1 = (56795800000 - 567958) = 56795232042. (ii) 978 x 184 + 978 x 816 = 978 x (184 + 816) = 978 x 1000 = 978000.

2. Multiplication of a Number By 5^n : Put n zeros to the right of the multiplicand and divide the number so formed by 2^n

Ex. 975436 x 625 = 975436 x 5^4= 9754360000 = 609647600

VII. BASIC FORMULAE

1. $(a + b)^2 = a^2 + b^2 + 2ab$ 2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 - (a - b)^2 = 4ab$ 4. $(a + b)^2 + (a - b)^2 = 2 (a^2 + b^2)$
5. $(a^2 - b^2) = (a + b) (a - b)$
6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2 (ab + bc + ca)$
7. $(a^3 + b^3) = (a + b) (a^2 - ab + b^2)$ 8. $(a^3 - b^3) = (a - b) (a^2 + ab + b^2)$
9. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$
10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

VIII. DIVISION ALGORITHM OR EUCLIDEAN ALGORITHM

If we divide a given number by another number, then :

Dividend = (Divisor x Quotient) + Remainder

IX. {i) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.
(ii) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.
(iii) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

X. PROGRESSION

A succession of numbers formed and arranged in a definite order according to certain definite rule, is called a progression.

1. Arithmetic Progression (A.P.) : If each term of a progression differs from its preceding term by a constant, then such a progression is called an arithmetical progression. This constant difference is called the *common difference* of the A.P.

An A.P. with first term a and common difference d is given by a, (a + d), (a + 2d),(a + 3d),.....

The **nth term of this A.P. is given by T_n =a (n - 1) d.**

The sum of n terms of this A.P.

$S_n = n/2\ [2a + (n - 1) d] = n/2$ (first term + last term).

SOME IMPORTANT RESULTS :

(i) $(1 + 2 + 3 +\dots + n) = n(n+1)/2$
(ii) $(l^2 + 2^2 + 3^2 + ... + n^2) = n (n+1)(2n+1)/6$
(iii) $(1^3 + 2^3 + 3^3 + ... + n^3) = n^2(n+1)^2$

2. Geometrical Progression (G.P.) : A progression of numbers in which every term bears a constant ratio with its preceding term, is called a geometrical progression.

The constant ratio is called the common ratio of the G.P. A G.P. with first term a and common ratio r is :

a, ar, ar^2,

In this G.P. $T_n = ar^{n-1}$

sum of the n terms, Sn= $\dfrac{a(1-r^n)}{(1-r)}$

SOLVED EXAMPLES

Ex. 1. Simplify : (i) 8888 + 888 + 88 + 8
(ii) 11992 - 7823 - 456

Sol. i) 8888 + 888 + 88 + 8 = 9872

$$\begin{array}{r} 8888 \\ 888 \\ 88 \\ +\quad 8 \\ \hline 9872 \\ \hline \end{array}$$

ii) 11992 - 7823 - 456 = 11992 - (7823 + 456)
= 11992 - 8279 = 3713-

$$\begin{array}{r} 7823 \\ +\ 456 \\ \hline 8279 \\ \hline \end{array} \qquad \begin{array}{r} 11992 \\ -\ 8279 \\ \hline 3713 \\ \hline \end{array}$$

Ex. 2, What value will replace the question mark in each of the following equations ?
(i) ? - 1936248 = 1635773 (ii) 8597 - ? = 7429 - 4358

Sol. (i) Let x - 1936248=1635773.Then, x = 1635773 + 1936248=3572021.
(ii) Let 8597 - x = 7429 - 4358.
Then, x = (8597 + 4358) - 7429 = 12955 - 7429 = 5526.

Ex. 3. What could be the maximum value of Q in the following equation? 5P9 + 3R7 + 2Q8 = 1114

Sol. We may analyse the given equation as shown :
Clearly, 2 + P + R + Q = ll.
So, the maximum value of Q can be
(11 - 2) i.e., 9 (when P = 0, R = 0);

$$\begin{array}{rrr} 1 & 2 & \\ 5 & P & 9 \\ 3 & R & 7 \\ 2 & Q & 8 \\ \hline 11 & 1 & 4 \\ \hline \end{array}$$

Ex. 4. Simplify : (i) 5793405 x 9999 (ii) 839478 x 625
Sol.
i)5793405x9999=5793405(10000-1)=57934050000-5793405=57928256595.b

ii) 839478 x 625 = 839478 x 5^4 = $\frac{8394780000}{16}$ = 524673750.

Ex. 5. Evaluate : (i) 986 x 237 + 986 x 863 (ii) 983 x 207 - 983 x 107
Sol.
(i) 986 x 137 + 986 x 863 = 986 x (137 + 863) = 986 x 1000 = 986000.
(ii) 983 x 207 - 983 x 107 = 983 x (207 - 107) = 983 x 100 = 98300.

Ex. 6. Simplify : (i) 1605 x 1605 ii) 1398 x 1398
Sol.
i) 1605 x 1605 = $(1605)^2 = (1600 + 5)^2 = (1600)^2 + (5)^2 + 2 \times 1600 \times 5$
= 2560000 + 25 + 16000 = 2576025.
(ii) 1398 x 1398 - $(1398)^2 = (1400 - 2)^2 = (1400)^2 + (2)^2 - 2 \times 1400 \times 2$

=1960000 + 4 - 5600 = 1954404.

Ex. 7. Evaluate : (313 x 313 + 287 x 287).
Sol.
$(a^2 + b^2) = 1/2\ [(a + b)^2 + (a - b)^2]$
$(313)^2 + (287)^2 = 1/2\ [(313 + 287)^2 + (313 - 287)^2] = ½[(600)^2 + (26)^2]$
= 1/2 (360000 + 676) = 180338.

Ex. 8. Which of the following are prime numbers ?
(i) 241 (ii) 337 (Hi) 391 (iv) 571
Sol.
(i) Clearly, 16 > Ö241. Prime numbers less than 16 are 2, 3, 5, 7, 11, 13.
241 is not divisible by any one of them.
241 is a prime number.

(ii) Clearly, 19>Ö337. Prime numbers less than 19 are 2, 3, 5, 7, 11,13,17.
337 is not divisible by any one of them.
337 is a prime number.
(iii) Clearly, 20 > Ö39l". Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
We find that 391 is divisible by 17.
391 is not prime.
(iv) Clearly, 24 > Ö57T. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23.
571 is not divisible by any one of them.
571 is a prime number.

Ex. 9. Find the unit's digit in the product (2467)163 x (341)72.
Sol. Clearly, unit's digit in the given product = unit's digit in 7^{153} x 1^{72}.
Now, 74 gives unit digit 1.
7^{152} gives unit digit 1,
$\therefore$ 7^{153} gives unit digit (l x 7) = 7. Also, 1^{72} gives unit digit 1.
Hence, unit's digit in the product = (7 x 1) = 7.

Ex. 10. Find the unit's digit in $(264)^{102} + (264)^{103}$
Sol. Required unit's digit = unit's digit in $(4)^{102} + (4)^{103.}$
Now, 4^2 gives unit digit 6.
$\therefore (4)^{102}$ gives unjt digit 6.
$\therefore$ (4)103 gives unit digit of the product (6 x 4) i.e., 4.
Hence, unit's digit in (264)m + (264)103 = unit's digit in (6 + 4) = 0.

Ex. 11. Find the total number of prime factors in the expression $(4)^{11}$ x $(7)^5$ x $(11)^2$.
Sol. $(4)^{11}$x $(7)^5$ x $(11)^2 = (2 \text{ x } 2)^{11}$ x $(7)^5$ x $(11)^2 = 2^{11}$ x 2^{11} x7^5x $11^2 = 2^{22}$ x 7^5 x11^2
Total number of prime factors = (22 + 5 + 2) = 29.

Ex.12. Simplify : (i) 896 x 896 - 204 x 204
(ii) 387 x 387 + 114 x 114 + 2 x 387 x 114
(iii) 81 X 81 + 68 X 68-2 x 81 X 68.

Sol.

(i) Given exp = $(896)^2 - (204)^2 = (896 + 204)(896 - 204) = 1100 \times 692 = 761200$.

(ii) Given exp $= (387)^2 + (114)^2 + (2 \times 387 \times 114)$
$= a^2 + b^2 + 2ab$, where $a = 387, b=114$
$= (a+b)^2 = (387 + 114)^2 = (501)^2 = 251001$.

(iii) Given exp $= (81)^2 + (68)^2 - 2 \times 81 \times 68 = a^2 + b^2 - 2ab$, Where $a = 81, b=68$
$= (a-b)^2 = (81 - 68)^2 = (13)^{2 = 169.}$

Ex.13. Which of the following numbers is divisible by 3 ?
(i) 541326 (ii) 5967013

Sol.

(i) Sum of digits in 541326 = (5 + 4 + 1 + 3 + 2 + 6) = 21, which is divisible by 3.
Hence, 541326 is divisible by 3.

(ii) Sum of digits in 5967013 =(5+9 + 6 + 7 + 0+1 +3) = 31, which is not divisible by 3.
Hence, 5967013 is not divisible by 3.

Ex.14.What least value must be assigned to * so that the number 197*5462 is r 9 ?

Sol.

Let the missing digit be x.
Sum of digits = (1 + 9 + 7 + x + 5 + 4 + 6 +»2) = (34 + x).
For (34 + x) to be divisible by 9, x must be replaced by 2 .
Hence, the digit in place of * must be 2.

Ex. 15. Which of the following numbers is divisible by 4 ?
(i) 67920594 (ii) 618703572

Sol.

(i) The number formed by the last two digits in the given number is 94, which is not divisible by 4.
Hence, 67920594 is not divisible by 4.

(ii) The number formed by the last two digits in the given number is 72, which is divisible by 4.
Hence, 618703572 is divisible by 4.

Ex. 16. Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 and 5 ?

Sol.

Since the given number is divisible by 5, so 0 or 5 must come in place of $. But, a number ending with 5 is never divisible by 8. So, 0 will replace $.

Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.

Hence, digits in place of * and $ are 4 and 0 respectively.

Ex. 17. Show that 4832718 is divisible by 11.

Sol. (Sum of digits at odd places) - (Sum of digits at even places)

$= (8 + 7 + 3 + 4) - (1 + 2 + 8) = 11$, which is divisible by 11.

Hence, 4832718 is divisible by 11.

Ex. 18. Is 52563744 divisible by 24 ?

Sol. $24 = 3 \times 8$, where 3 and 8 are co-primes.

The sum of the digits in the given number is 36, which is divisible by 3. So, the given number is divisible by 3.

The number formed by the last 3 digits of the given number is 744, which is divisible by 8. So, the given number is divisible by 8.

Thus, the given number is divisible by both 3 and 8, where 3 and 8 are co-primes. So, it is divisible by 3×8, i.e., 24.

Ex. 19. What least number must be added to 3000 to obtain a number exactly divisible by 19 ?

Sol. On dividing 3000 by 19, we get 17 as remainder.

$\therefore$ Number to be added $= (19 - 17) = 2$.

Ex. 20. What least number must be subtracted from 2000 to get a number exactly divisible by 17 ?

Sol. On dividing 2000 by 17, we get 11 as remainder.

$\therefore$ Required number to be subtracted $= 11$.

Ex. 21. Find the number which is nearest to 3105 and is exactly divisible by 21.

Sol. On dividing 3105 by 21, we get 18 as remainder.

$\therefore$ Number to be added to 3105 $= (21 - 18) - 3$.

Hence, required number $= 3105 + 3 = 3108$.

Ex. 22. Find the smallest number of 6 digits which is exactly divisible by 111.

Sol. Smallest number of 6 digits is 100000.

On dividing 100000 by 111, we get 100 as remainder.

$\therefore$ Number to be added = (111 - 100) - 11.

Hence, required number = 100011.-

Ex. 23. On dividing 15968 by a certain number, the quotient is 89 and the remainder is 37. Find the divisor.

Sol. $$\text{Divisor} = \frac{\text{Dividend - Remainder}}{\text{.Quotient}} = \frac{15968-37}{89} = 179.$$

Ex. 24. A number when divided by 342 gives a remainder 47. When the same number ift divided by 19, what would be the remainder ?

Sol. On dividing the given number by 342, let k be the quotient and 47 as remainder.

Then, number – 342k + 47 = (19 x 18k + 19 x 2 + 9) = 19 (18k + 2) + 9.

$\therefore$ The given number when divided by 19, gives (18k + 2) as quotient and 9 as remainder.

Ex. 25. A number being successively divided by 3, 5 and 8 leaves remainders 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed,

Sol.

3	X	
5	y	- 1
8	z	- 4
	1	- 7

$\therefore$ z = (8 x 1 + 7) = 15; y = {5z + 4) = (5 x 15 + 4) = 79; x = (3y + 1) = (3 x 79 + 1) = 238.

Now,

8	238	
5	29	- 6
3	5	- 4
	1	- 9,

$\therefore$ Respective remainders are 6, 4, 2.

Ex. 26. Find the remainder when 2^{31} is divided by 5.

Sol. 2^{10} = 1024. Unit digit of 2^{10} x 2^{10} x 2^{10} is 4 [as 4 x 4 x 4 gives unit digit 4].

$\therefore$ Unit digit of 231 is 8.

Now, 8 when divided by 5, gives 3 as remainder.

Hence, 231 when divided by 5, gives 3 as remainder.

Ex. 27. How many numbers between 11 and 90 are divisible by 7 ?

Sol. The required numbers are 14, 21, 28, 35, ... 77, 84.

This is an A.P. with a = 14 and d = (21 - 14) = 7.

Let it contain n terms.

Then, $T_n = 84 \Rightarrow a + (n - 1) d = 84$

$\Rightarrow 14 + (n - 1) \times 7 = 84$ or n = 11.

$\therefore$ Required number of terms = 11.

Ex. 28. Find the sum of all odd numbers upto 100.

Sol. The given numbers are 1, 3, 5, 7, ..., 99.

This is an A.P. with a = 1 and d = 2.

Let it contain n terms. Then,

1 + (n - 1) x 2 = 99 or n = 50.

$\therefore$ Required sum $= \frac{n}{2}$ (first term + last term)

$= \frac{50}{2}(1 + 99) = 2500.$

Ex. 29. Find the sum of all 2 digit numbers divisible by 3.

Sol. All 2 digit numbers divisible by 3 are :

12, 51, 18, 21, ..., 99.

This is an A.P. with a = 12 and d = 3.

Let it contain n terms. Then,

12 + (n - 1) x 3 = 99 or n = 30.

$\therefore$ Required sum $= \frac{30}{2} \times (12+99) = 1665.$

Ex.30.How many terms are there in 2,4,8,16……1024?

Sol. Clearly 2,4,8,16……..1024 form a GP. With a=2 and r = 4/2 =2.

Let the number of terms be n . Then

$2 \times 2^{n-1} = 1024$ or $2^{n-1 = 512 = 2}9$.

$\therefore$ n-1=9 or n=10.

Ex. 31. $2 + 2^2 + 2^3 + ... + 2^8 = ?$

Sol. Given series is a G.P. with a = 2, r = 2 and n = 8.

$\therefore$ sum $= \frac{a(r^n-1)}{(r-1)} = \frac{2 \times (2^8 - 1)}{(2-1)} = (2 \times 255) = 510$

2. H.C.F. AND L.C.M. OF NUMBERS

IMPORTANT FACTS AND FORMULAE

I. Factors and Multiples : If a number a divides another number b exactly, we say that a is a factor of b. In this case, b is called a multiple of a.

II. Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.): The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are *two methods* of finding the H.C.F. of a given set of numbers :

1. Factorization Method : Express each one of the given numbers as the product of prime factors.The product of least powers of common prime factors gives H.C.F.

2. Division Method: Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.

Finding the H.C.F. of more than two numbers : Suppose we have to find the H.C.F. of three numbers. Then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.

Similarly, the H.C.F. of more than three numbers may be obtained.

III. Least Common Multiple (L.C.M.) : The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

1. Factorization Method of Finding L.C.M.: Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors,

2. Common Division Method {Short-cut Method) of Finding L.C.M.: Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers,

IV. Product of two numbers =Product of their H.C.F. and L.C.M.

V. Co-primes: Two numbers are said to be co-primes if their H.C.F. is 1.

VI. H.C.F. and L.C.M. of Fractions:

1. $\text{H C F} = \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$ 2. $\text{L C M} = \dfrac{\text{L.C.M of Numerators}}{\text{H.C.F. of Denominators}}$

VII. H.C.F. and L.C.M. of Decimal Fractions: In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

VIII. Comparison of Fractions: Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

SOLVED EXAMPLES

Ex. 1. Find the H.C.F. of 2^3 X 3^2 X 5 X 7^4, 2^2 X 3^5 X 5^2 X 7^3,2^3 X 5^3 X 7^2
Sol. The prime numbers common to given numbers are 2,5 and 7.
H.C.F. = 2^2 x 5 x7^2 = 980.

Ex. 2. Find the H.C.F. of 108, 288 and 360.
Sol. 108 = 2^2 x 3^3, 288 = 2^5 x 3^2 and 360 = 2^3 x 5 x 3^2.
H.C.F. = 2^2 x 3^2 = 36.

Ex. 3. Find the H.C.F. of 513, 1134 and 1215.
Sol.

```
1134 ) 1215 ( 1
        1134
        ------
          81 ) 1134 ( 14
                 81
                 324
                 324
                 ---
                  x
```

∴H.C.F. of 1134 and 1215 is 81.
So, Required H.C.F. = H.C.F. of 513 and 81.

```
81 )  513 ( 6
      486
      -----
       27) 81 ( 3
           81
           --
            0
```

H.C.F. of given numbers = 27.

Ex. 4. Reduce $\frac{391}{667}$ to lowest terms.
to lowest terms.

Sol. H.C.F. of 391 and 667 is 23.
On dividing the numerator and denominator by 23, we get :
$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}$

Ex.5.Find the L.C.M. of 2^2 x 3^3 x 5 x 7^2, 2^3 x 3^2 x 5^2 x 7^4 , 2 x 3 x $5^{3 \text{ x } 7 \text{ x } 11.}$
Sol. L.C.M. = Product of highest powers of 2, 3, 5, 7 and 11 = 2^3 x 3^3 x 5^3 x 7^4 x 11

Ex.6. Find the L.C.M. of 72, 108 and 2100.
Sol. 72 = 2^3 x 3^2, 108 = 3^3 x 2^2, 2100 = 2^2 x 5^2 x 3 x 7.
L.C.M. = 2^3 x 3^3 x 5^2 x 7 = 37800.

Ex.7.Find the L.C.M. of 16, 24, 36 and 54.

Sol.

2	16 - 24 - 36 - 54
2	8 - 12 - 18 - 27
2	4 - 6 - 9 - 27
3	2 - 3 - 9 - 27
3	2 - 1 - 3 - 9
	2 - 1 - 1 - 3

$\therefore$ L.C.M. = 2 x 2 x 2 x 3 x 3 x 2 x 3 = 432.

Ex. 8. Find the H.C.F. and L.C.M. of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$.

Sol. H.C.F. of given fractions = $\frac{\text{H.C.F. of } 2,8,16,10}{\text{L.C.M. of } 3,9,81,27} = \frac{2}{81}$

L.C.M of given fractions = $\frac{\text{L.C.M. of } 2,8,16,10}{\text{H.C.F. of } 3,9,81,27} = \frac{80}{3}$

Ex. 9. **Find the H.C.F. and L.C.M. of 0.63, 1.05 and 2.1.**

Sol. Making the same number of decimal places, the given numbers are 0.63, 1.05 and 2.10.

Without decimal places, these numbers are 63, 105 and 210.

Now, H.C.F. of 63, 105 and 210 is 21.

H.C.F. of 0.63, 1.05 and 2.1 is 0.21.

L.C.M. of 63, 105 and 210 is 630.

L.C.M. of 0.63, 1.05 and 2.1 is 6.30.

Ex. 10. Two numbers are in the ratio of 15:11. If their H.C.F. is 13, find the numbers.

Sol. Let the required numbers be 15.x and llx.

Then, their H.C.F. is x. So, x = 13.

The numbers are (15 x 13 and 11 x 13) i.e., 195 and 143.

Ex. 11. TheH.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77,find the other.

Sol. Other number = $\frac{11 \text{ X } 693}{77} = 99$

Ex. 12. Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.

Sol. Required length = H.C.F. of 495 cm, 900 cm and 1665 cm.

$495 = 3^2 \times 5 \times 11$, $900 = 2^2 \times 3^2 \times 5^2$, $1665 = 3^2 \times 5 \times 37$.

$\therefore$ H.C.F. = 32 x 5 = 45.

Hence, required length = 45 cm.

Ex. 13. Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.

Sol. Required number = H.C.F. of (1657 - 6) and (2037 - 5) = H.C.F. of 1651 and 2032

1651) 2032 (1 1651
1651
381) 1651 (4
1524
127) 381 (3
381
0

Required number = 127.

Ex. 14. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.

Sol . Required number = H.C.F. of (132 - 62), (237 - 132) and (237 - 62)

= H.C.F. of 70, 105 and 175 = 35.

Ex.15.Find the least number exactly divisible by 12,15,20,27.

Sol.

3	12 - 15 - 20 - 27
4	4 - 5 - 20 - 9
5	1 - 5 - 5 - 9
	1 - 1 - 1 - 9

Ex.16.Find the least number which when divided by 6,7,8,9, and 12 leave the same remainder 1 each case
Sol. Required number = (L.C.M OF 6,7,8,9,12) + 1

3	6	- 7	- 8	- 9	- 12
4	2	- 7	- 8	- 3	- 4
5	1	- 7	- 4	- 3	- 2
	1	- 7	- 2	- 3	- 1

$\therefore$L.C.M = 3 X 2 X 2 X 7 X 2 X 3 = 504.
Hence required number = (504 +1) = 505.

Ex.17. Find the largest number of four digits exactly divisible by 12,15,18 and 27.
Sol. The Largest number of four digits is 9999.
Required number must be divisible by L.C.M. of 12,15,18,27 i.e. 540.
On dividing 9999 by 540,we get 279 as remainder .
$\therefore$Required number = (9999-279) = 9720.

Ex.18.Find the smallest number of five digits exactly divisible by 16,24,36 and 54.
Sol. Smallest number of five digits is 10000.
Required number must be divisible by L.C.M. of 16,24,36,54 i.e 432,
On dividing 10000 by 432,we get 64 as remainder.
$\therefore$Required number = 10000 +(432 – 64) = 10368.

Ex.19.Find the least number which when divided by 20,25,35 and 40 leaves remainders 14,19,29 and 34 respectively.
Sol. Here,(20-14) = 6,(25 – 19)=6,(35-29)=6 and (40-34)=6.
$\therefore$Required number = (L.C.M. of 20,25,35,40) – 6 =1394.

Ex.20.Find the least number which when divided by 5,6,7, and 8 leaves a remainder 3, but when divided by 9 leaves no remainder .
Sol. L.C.M. of 5,6,7,8 = 840.
$\therefore$ Required number is of the form 840k + 3
Least value of k for which (840k + 3) is divisible by 9 is k = 2.
$\therefore$Required number = (840 X 2 + 3)=1683

Ex.21.The traffic lights at three different road crossings change after every 48 sec., 72 sec and 108 sec.respectively .If they all change simultaneously at 8:20:00 hours,then at what time they again change simultaneously .
Sol. Interval of change = (L.C.M of 48,72,108)sec.=432sec.
So, the lights will agin change simultaneously after every 432 seconds i.e,7 min.12sec
Hence , next simultaneous change will take place at 8:27:12 hrs.

Ex.22. Arrange the fractions $\frac{17}{18}, \frac{31}{36}, \frac{43}{45}, \frac{59}{60}$ in the ascending order.

Sol. L.C.M. of 18,36,45 and 60 = 180.

Now, $\frac{17}{18} = \frac{17 \times 10}{18 \times 10} = \frac{170}{180}$; $\frac{31}{36} = \frac{31 \times 5}{36 \times 5} = \frac{155}{180}$;

$\frac{43}{45} = \frac{43 \times 4}{45 \times 4} = \frac{172}{180}$; $\frac{59}{60} = \frac{59 \times 3}{60 \times 3} = \frac{177}{180}$;

Since, $155<170<172<177$, so, $\frac{155}{180} < \frac{170}{180} < \frac{172}{180} < \frac{177}{180}$

Hence, $\frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$

3. DECIMAL FRACTIONS

IMPORTANT FACTS AND FORMULAE

I. Decimal Fractions : Fractions in which denominators are powers of 10 are known as **decimal fractions.**

Thus ,1/10=1 tenth=.1;1/100=1 hundredth =.01;

99/100=99 hundreths=.99;7/1000=7 thousandths=.007,etc

II. Conversion of a Decimal Into Vulgar Fraction : Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Now, remove the decimal point and reduce the fraction to its lowest terms.

Thus, 0.25=25/100=1/4;2.008=2008/1000=251/125.

III. 1. Annexing zeros to the extreme right of a decimal fraction does not change its value
Thus, 0.8 = 0.80 = 0.800, etc.

2. If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.
Thus, 1.84/2.99 = 184/299 = 8/13; 0.365/0.584 = 365/584=5

IV. Operations on Decimal Fractions :

1. Addition and Subtraction of Decimal Fractions : The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in the usual way.

2. **Multiplication of a Decimal Fraction By a Power of 10** : Shift the decimal point to the right by as many places as is the power of 10.
Thus, 5.9632 x 100 = 596,32; 0.073 x 10000 = 0.0730 x 10000 = 730.

3.**Multiplication of Decimal Fractions** : Multiply the given numbers considering them without the decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.
Suppose we have to find the product (.2 x .02 x .002). Now, 2x2x2 = 8. Sum of decimal places = (1 + 2 + 3) = 6. .2 x .02 x .002 = .000008.

4.**Dividing a Decimal Fraction By a Counting Number :** Divide the given number without considering the decimal point, by the given counting number. Now, in the quotient, put the decimal point to give as many places of decimal as there are in the dividend.
Suppose we have to find the quotient (0.0204 + 17). Now, 204 ^ 17 = 12. Dividend contains 4 places of decimal. So, 0.0204 + 17 = 0.0012.

5. **Dividing a Decimal Fraction By a Decimal Fraction :** Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.

Thus, 0.00066/0.11 = (0.00066*100)/(0.11*100) = (0.066/11) = 0.006V

V. Comparison of Fractions : Suppose some fractions are to be arranged in ascending or descending order of magnitude. Then, convert each one of the given fractions in the decimal form, and arrange them accordingly.

Suppose, we have to arrange the fractions 3/5, 6/7 and 7/9 in descending order.

now, 3/5=0.6,6/7 = 0.857,7/9 = 0.777....

since 0.857>0.777...>0.6, so 6/7>7/9>3/5

VI. Recurring Decimal : If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a *recurring decimal.*

In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set

Thus 1/3 = 0.3333….= 0.3; 22 /7 = 3.142857142857.....= $3.\overline{142857}$

Pure Recurring Decimal: A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal Into Vulgar Fraction : Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

thus ,0.5 = 5/9; 0.53 = 53/59 ;0.067 = 67/999;etc...

Mixed Recurring Decimal: A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.

e.g., 0.17333.= 0.173.

Converting a Mixed Recurring Decimal Into Vulgar Fraction : In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated, In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Thus 0.16 = (16-1) / 90 = 15/19 = 1/6;

$0.\overline{2273}$ = (2273 – 22)/9900 = 2251/9900

VII. Some Basic Formulae :

1. $(a + b)(a - b) = (a^2 - b^2)$.
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$.
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$.
4. $(a + b+c)^2 = a^2 + b^2 + c^2 +2(ab+bc+ca)$
5. $(a^3 + b^3) = (a + b)\,(a^2 - ab + b^2)$
6. $(a^3 - b^3) = (a - b)\,(a^2 + ab + b^2)$.
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)\,(a^2 + b^2 + c^2-ab-bc-ca)$
8. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

SOLVED EXAMPLES

Ex. 1. Convert the following into vulgar fraction:
(i) 0.75 (ii) 3.004 (iii) 0.0056

Sol. (i). 0.75 = 75/100 = 3/4 (ii) 3.004 = 3004/1000 = 751/250 (iii) 0.0056 = 56/10000 = 7/1250

Ex. 2. Arrange the fractions 5/8, 7/12, 13/16, 16/29 and 3/4 in ascending order of magnitude.

Sol. Converting each of the given fractions into decimal form, we get :
5/8 = 0.624, 7/12 = 0.8125, 16/29 = 0.5517, and 3/4 = 0.75
Now, 0.5517<0.5833<0.625<0.75<0.8125
$\therefore$ 16/29 < 7/12 < 5/8 < 3/4 < 13/16

Ex. 3. arrange the fractions 3/5, 4/7, 8/9, and 9/11 in their descending order.

Sol. Clearly, 3/5 = 0.6, 4/7 = 0.571, 8/9 = 0.88, 9/111 = 0.818.
Now, 0.88 > 0.818 > 0.6 > 0.571
$\therefore$ 8/9 > 9/11 > 3/4 > 13/ 16

Ex. 4. Evaluate : (i) 6202.5 + 620.25 + 62.025 + 6.2025 + 0.62025
(ii) 5.064 + 3.98 + 0.7036 + 7.6 + 0.3 + 2

Sol. (i)

$$\begin{array}{r} 6202.5 \\ 620.25 \\ 62.025 \\ 6.2025 \\ +\ \underline{0.62025} \\ \underline{6891.59775} \end{array}$$

(ii)

$$\begin{array}{l} 5.064 \\ 3.98 \\ 0.7036 \\ 7.6 \\ 0.3 \\ \underline{2.0} \\ \underline{19.6476} \end{array}$$

Ex. 5. Evaluate : (i) 31.004 – 17.2368 (ii) 13 – 5.1967

Sol. (i) 31.0040 − 17.2386 = 13.7654   (ii) 31.0000 − 5.1967 = 7.8033

$$\begin{array}{r} 31.0040 \\ -\ \underline{17.2386} \\ 13.7654 \end{array} \qquad \begin{array}{r} 31.0000 \\ -\ \underline{\ \ 5.1967} \\ 7.8033 \end{array}$$

Ex. 6. What value will replace the question mark in the following equations ?

(i) 5172.49 + 378.352 + ? = 9318.678

(ii) ? – 7328.96 + 5169.38

Sol. (i) Let 5172.49 + 378.352 + x = 9318.678

Then , x = 9318.678 – (5172.49 + 378.352) = 9318.678 – 5550.842 = 3767.836

(ii) Let x – 7328.96 = 5169.38. Then, x = 5169.38 + 7328.96 = 12498.34.

Ex. 7. Find the products: (i) 6.3204 * 100 (ii) 0.069 * 10000

Sol. (i) 6.3204 * 1000 = 632.04   (ii) 0.069 * 10000 = 0.0690 * 10000 = 690

Ex. 8. Find the product:

(i) 2.61 * 1.3   (ii) 2.1693 * 1.4   (iii) 0.4 * 0.04 * 0.004 * 40

Sol. (i) 261 8 13 = 3393. Sum of decimal places of given numbers = (2+1) = 3.

2.61 * 1.3 = 3.393.

(ii) 21693 * 14 = 303702. Sum of decimal places = (4+1) = 5

2.1693 * 1.4 = 3.03702.

(iii) 4 * 4 * 4 * 40 = 2560. Sum of decimal places = (1 + 2+ 3) = 6

0.4 * 0.04 * 0.004 * 40 = 0.002560.

Ex. 9. Given that 268 * 74 = 19832, find the values of 2.68 * 0.74.

Sol. Sum of decimal places = (2 + 2) = 4

2.68 * 0.74 = 1.9832.

Ex. 10. Find the quotient:

(i) 0.63 / 9   (ii) 0.0204 / 17   (iii) 3.1603 / 13

Sol. (i) 63 / 9 = 7. Dividend contains 2 places decimal.

0.63 / 9 = 0.7.

(ii) 204 / 17 = 12. Dividend contains 4 places of decimal.

0.2040 / 17 = 0.0012.

(iii) 31603 / 13 = 2431. Dividend contains 4 places of decimal.

3.1603 / 13 = 0.2431.

Ex. 11. Evaluate :

(i) 35 + 0.07   (ii) 2.5 + 0.0005

(iii) 136.09 + 43.9

Sol. (i) 35/0.07 = (35*100) / (0.07*100) = (3500 / 7) = 500
(ii) 25/0.0005 = (25*10000) / (0.0005*10000) = 25000 / 5 = 5000
(iii) 136.09/43.9 = (136.09*10) / (43.9*10) = 1360.9 / 439 = 3.1

Ex. 12. What value will come in place of question mark in the following equation?
(i) 0.006 +? = 0.6 (ii) ? + 0.025 = 80

Sol. (i) Let 0.006 / x = 0.6, Then, x = (0.006 / 0.6) = (0.006*10) / (0.6*10) = 0.06/6 = 0.01
(ii) Let x / 0.025 = 80, Then, x = 80 * 0.025 = 2

Ex. 13. If (1 / 3.718) = 0.2689, Then find the value of (1 / 0.0003718).

Sol. (1 / 0.0003718) = (10000 / 3.718) = 10000 * (1 / 3.718) = 10000 * 0.2689 = 2689.

Ex. 14. Express as vulgar fractions : (i) $0.\overline{37}$ (ii) $0.\overline{053}$ (iii) $3.\overline{142857}$

Sol. (i) $0.\overline{37}$ = 37 / 99 . (ii) $0.\overline{053}$ = 53 / 999

(iii) $3.\overline{142857}$ = 3 + $0.\overline{142857}$ = 3 + (142857 / 999999) = 3 (142857/999999)

Ex. 15. Express as vulgar fractions : (i) $0.1\overline{7}$ (ii) $0.12\overline{54}$ (iii) $2.53\overline{6}$

Sol. (i) $0.1\overline{7}$ = (17 – 1)/90 = 16 / 90 = 8/ 45

(ii) $0.12\overline{54}$ = (1254 – 12)/ 9900 = 1242 / 9900 = 69 / 550

(iii) 2.536 = 2 + 0.536 = 2 + (536 – 53)/900 = 2 + (483/900) = 2 + (161/300) = 2 (161/300)

Ex. 16. Simplify:

$$\frac{0.05 * 0.05 * 0.05 + 0.04 * 0.04 * 0.04}{0.05 * 0.05 - 0.05 * 0.04 + 0.04 * 0.04}$$

Sol. Given expression = $(a^3 + b^3) / (a^2 - ab + b^2)$, where a = 0.05 , b = 0.04
= (a +b) = (0.05 +0.04) =0.09

4. SIMPLIFICATION

IMPORTANT CONCEPTS

I. 'BODMAS'Rule: This rule depicts the correct sequence in which the operations are to be executed,so as to find out the value of a given expression.

Here, 'B' stands for 'bracket' ,'O'for 'of' , 'D' for' division' and 'M' for 'multiplication', 'A' for 'addition' and 'S' for 'subtraction'.
Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order(), { } and [].

After removing the brackets, we must use the following operations strictly in the order:
(1)of (2)division (3) multiplication (4)addition (5)subtraction.

II. Modulus of a real number : Modulus of a real number a is defined as

$$|a| = \begin{cases} a, & \text{if } a>0 \\ -a, & \text{if } a<0 \end{cases}$$

Thus, $|5|=5$ and $|-5|=-(-5)=5$.
III. Virnaculum (or bar): When an expression contains Virnaculum, before applying the 'BODMAS' rule, we simplify the expression under the Virnaculum.

SOLVED EXAMPLES

Ex. 1. Simplify: (i)5005-5000+10 (ii) 18800+470+20

Sol. (i)5005-5000+10=5005-(5000/10)=5005-500=4505.

(ii)18800+470+20=(18800/470)+20=40/20=2.

Ex. 2. Simplify: b-[b-(a+b)-{b-(b-a-b)}+2a]

Sol. Given expression=b-[b-(a+b)-{b-(b-a+b)}+2a]
=b-[b-a-b-{b-2b+a}+2a]
=b-[-a-{b-2b+a+2a}]
=b-[-a-{-b+3a}]=b-[-a+b-3a]
=b-[-4a+b]=b+4a-b=4a.

Ex. 3. What value will replace the question mark in the following equation?

$$4\frac{1}{2}+3\frac{1}{6}+?+2\frac{1}{3}=13\frac{2}{5}.$$

Sol. Let 9/2+19/6+x+7/3=67/5

Then $x=(67/5)-(9/2+19/6+7/3) \Leftrightarrow x=(67/5)-((27+19+14)/6)=((67/5)-(60/6)$
$\Leftrightarrow x=((67/5)-10)=17/5=3\frac{2}{5}$

Hence, missing fractions $=3\frac{2}{5}$

Ex.4. 4/15 of 5/7 of a number is greater than 4/9 of 2/5 of the same number by 8. What is half of that number?

Sol. Let the number be x. then 4/15 of 5/7 of x-4/9 of 2/5 of $x=8 \Leftrightarrow 4/21x-8/45x=8$
$\Leftrightarrow (4/21-8/45)x=8 \Leftrightarrow (60-56)/315x=8 \Leftrightarrow 4/315x=8$
$\Leftrightarrow x=(8*315)/4=630 \Leftrightarrow 1/2x=315$
Hence required number = 315.

Ex. 5. Simplify: $\left[3\frac{1}{4} \div \{1\frac{1}{4}-1/2(2\frac{1}{2}-\overline{1/4-1/6})\}\right]$

Sol. Given exp. =[13/4÷{5/4-1/2(5/2-(3-2)/12)}]=[13/4÷{5/4-1/2(5/2-1/12)}]
=[13/4÷{5/4-1/2((30-1)/12)}]=[13/4÷{5/4-29/24}]
=[13/4÷{(30-39)/24}]=[13/4÷1/24]=[(13/4)*24]=78

Ex. 6. Simplify: $108 \div 36 \text{ of } \frac{1}{4}+\frac{2}{5}*3\frac{1}{4}$

Sol. Given exp.= $108\div 9+\frac{2}{5}*\frac{13}{4} = \frac{108}{9}+\frac{13}{10} = 12+\frac{13}{10} = \left(\frac{133}{10}\right) = 13\frac{3}{10}$

Ex.7 Simplify: $\dfrac{(7/2)\div(5/2)*(3/2)}{(7/2)\div(5/2)\text{of }(3/2)} \div 5.25$

sol.
Given exp. $\dfrac{(7/2)\times(2/5)\times(3/2)}{(7/2)\div(15/4)} \div 5.25=\underline{(21/10)}\div(525/100)=(21/10)\times\underline{(15/14)}$

Ex. 8. Simplify: (i) 12.05*5.4+0.6 (ii) 0.6*0.6+0.6*0.6 (Bank P.O 2003)

Sol. (i) Given exp. = 12.05*(5.4/0.6) = (12.05*9) = 108.45
(ii) Given exp. = 0.6*0.6+(0.6*6) = 0.36+0.1 = 0.46

Ex. 9. Find the value of x in each of the following equation:
(i) [(17.28/x) / (3.6*0.2)] = 2
(ii) 3648.24 + 364.824 + x – 36.4824 = 3794.1696
(iii) $8.5 - \{ 5\frac{1}{2} - [7\frac{1}{2} + 2.8]/x\}*4.25/(0.2)^2 = 306$ (Hotel Management,1997)

Sol. (i) $(17.28/x) = 2*3.6*0.2 \Leftrightarrow x = (17.28/1.44) = (1728/14) = 12.$
(ii) (364.824/x) = (3794.1696 + 36.4824) – 3648.24 = 3830.652 – 3648.24 = 182.412.

$\Leftrightarrow$ x = (364.824/182.412) =2.

(iii) 8.5-{5.5-(7.5+(2.8/x))}*(4.25/0.04) = 306

$\Leftrightarrow$ 8.5-{5.5-{(7.5x+2.8)/x)}*(425/4) = 306

$\Leftrightarrow$ 8.5-{(5.5x-7.5x-2.8)/x}*(425/4) = 306

$\Leftrightarrow$ 8.5-{(-2x-2.8)/x}*106.25 = 306

$\Leftrightarrow$ 8.5-{(-212.5x-297.5)/x} = 306

$\Leftrightarrow$ (306-221)x = 297.5 $\Leftrightarrow$ x =(297.5/85) = 3.5.

Ex. 10. If (x/y)=(6/5), find the value $(x^2+y^2)/(x^2-y^2)$

Sol. $(x^2+y^2)/(x^2-y^2) = (x^2/y^2+1)/(x^2/y^2-1) = [(6/5)^2+1] / [(6/5)^2-1]$

$= [(36/25)+1] / [(36/25)-1] = (61*25)/(25*11) = 61/11$

Ex. 11. Find the value of $4 - \cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4}}}}$

Sol. Given exp. = $4 - \cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4}}}} = 4 - \cfrac{5}{1+\cfrac{1}{3+\cfrac{4}{9}}} = 4 - \cfrac{5}{1+\cfrac{1}{(31/9)}}$

$= 4 - \cfrac{5}{1+\cfrac{9}{31}} = 4 - \cfrac{5}{(40/31)} = 4 - (5*31)/40 = 4 - (31/8) = 1/8$

Ex. 12. If $\cfrac{2x}{1+\cfrac{1}{1+\cfrac{x}{1-x}}} = 1$ **., then find the value of x .**

Sol. We have : $\cfrac{2x}{1+\cfrac{1}{\cfrac{(1-x)-x}{1-x}}} = 1 \Leftrightarrow \cfrac{2x}{1+\cfrac{1}{[1/(1-x)]}} = 1 \Leftrightarrow \cfrac{2x}{1+(1-x)} = 1$

$\Leftrightarrow$ 2x = 2-x $\Leftrightarrow$ 3x = 2 $\Leftrightarrow$ x = (2/3).

Ex.13.(i)If a/b=3/4 and 8a+5b=22,then find the value of a.

(ii)if x/4-x-3/6=1,then find the value of x.

Sol. (i) (a/b)=3/4 $\Rightarrow$b=(4/3) a.

$\therefore$ 8a+5b=22 $\Rightarrow$ 8a+5*(4/3) a=22 $\Rightarrow$ 8a+(20/3) a=22

$\Rightarrow$44a = 66 $\Rightarrow$ a=(66/44)=3/2

(ii) (x /4)-((x-3)/6)=1$\Leftrightarrow$ (3x-2(x-3))/12 = 1 $\Leftrightarrow$ 3x-2x+6=12 $\Leftrightarrow$ x=6.

Ex.14.If 2x+3y=34 and ((x + y)/y)=13/8,then find the value of 5y+7x.

Sol. The given equations are:
2x+3y=34 …(i) and, ((x + y) /y)=13/8 ⇒ 8x+8y=13y ⇒ 8x-5y=0 …(ii)
Multiplying (i) by 5,(ii) by 3 and adding, we get : 34x=170 or x=5.
Putting x=5 in (i), we get: y=8.
∴ 5y+7x=((5*8)+(7*5))=40+35=75

Ex.15.If 2x+3y+z=55,x-y=4 and y - x + z=12,then what are the values of x , y and z?

Sol. The given equations are:
2x+3y+z=55 …(i); x + z - y=4 …(ii); y -x + z =12 …(iii)
Subtracting (ii) from (i), we get: x+4y=51 …(iv)
Subtracting (iii) from (i), we get: 3x+2y=43 …(v)
Multiplying (v) by 2 and subtracting (iv) from it, we get: 5x=35 or x=7.
Putting x=7 in (iv), we get: 4y=44 or y=11.
Putting x=7,y=11 in (i), we get: z=8.

Ex.16.Find the value of (1-(1/3))(1-(1/4))(1-(1/5))….(1-(1/100)).

Sol. Given expression = (2/3)*(3/4)*(4/5) *…….* (99/100) = 2/100 = 1/50.

Ex.17. Find the value of (1/(2*3))+(1/(3*4))+(1/(4*5))+(1/(5*6))+…..+ ((1/(9*10)).

Sol. Given expression=((1/2)-(1/3))+((1/3)-(1/4))+((1/4)-(1/5))+
((1/5)-(1/6))+….+ ((1/9)-(1/10))
=((1/2)-(1/10))=4/10 = 2/5.

Ex.18.Simplify: $99^{48}/_{49}$ * 245.

Sol. Given expression = (100-1/49) * 245=(4899/49) * 245 = 4899 * 5=24495.

Ex.19.A board 7ft. 9 inches long is divided into 3 equal parts . What is the length of each part?

Sol. Length of board=7ft. 9 inches=(7*12+9)inches=93 inches.
∴ Length of each part = (93/3) inches = 31 inches = 2ft. 7 inches

20.A man divides Rs. Among 5 sons,4daughters and 2 nephews .If each daughter receives four times as much as each nephews and each son receives five times as much as each nephews ,how much does each daughter receive?
Let the share of each nephews be Rs.x.
Then,share of each daughter=rs4x;share of each son=Rs.5x;
So,5*5x+4*4x+2*x=8600

25x+16x+2x=8600
=43x=8600
x=200;

21.A man spends 2/5 of his salary on house rent,3/10 of his salary on food and 1/8 of his salary on conveyence.if he has Rs.1400 left with him,find his expenditure on food and conveyence.
Part of salary left=1-(2/5+3/10+1/8)
Let the monthly salary be Rs.x
Then, 7/40 of x=1400
X=(1400*40/7)
=8600
Expenditure on food=Rs.(3/10*800)=Rs.2400
Expenditure on conveyence=Rs.(1/8*8000)=Rs.1000

22.A third of Arun's marks in mathematics exeeds a half of his marks in english by 80.if he got 240 marks In two subjects together how many marks did he got inh english?
Let Arun's marks in mathematics and english be x and y
Then 1/3x-1/2y=30
2x-3y=180……>(1)
x+y=240…….>(2)
solving (1) and (2)
x=180
and y=60

23.A tin of oil was 4/5full.when 6 bottles of oil were taken out and four bottles of oil were poured into it, it was ¾ full. how many bottles of oil can the tin contain?
Suppose x bottles can fill the tin completely
Then4/5x-3/4x=6-4
X/20=2
X=40
Therefore required no of bottles =40

24.if 1/8 of a pencil is black ½ of the remaining is white and the remaining 3 ½ is blue find the total length of the pencil?
Let the total length be xm
Then black part =x/8cm
The remaining part=(x-x/8)cm=7x/8cm
White part=(1/2 *7x/8)=7x/16 cm
Remaining part=(7x/8-7x/16)=7x/16cm
7x/16=7/2
x=8cm

25.in a certain office 1/3 of the workers are women ½ of the women are married and 1/3 of the married women have children if ¾ of the men are married and 2/3 of the married men have children what part of workers are without children?

Let the total no of workers be x
No of women =x/3
No of men =x-(x/3)=2x/3
No of women having children =1/3 of ½ ofx/3=x/18
No of men having children=2/3 of ¾ of2x/3=x/3
No of workers having children = x/8 +x/3=7x/18
Workers having no children=x-7x/18=11x/18=11/18 of all wprkers

26.a crate of mangoes contains one bruised mango for every thirty mango in the crate. If three out of every four bruised mango are considerably unsaleble and there are 12 unsaleable mangoes in the crate then how msny mango are there in the crate?

Let the total no of mangoes in the crate be x
Then the no of bruised mango = 1/30 x
Let the no of unsalable mangoes =3/4 (1/30 x)
1/40 x =12
x=480

27. a train starts full of passengers at the first station it drops 1/3 of the passengers and takes 280more at the second station it drops one half the new total and takes twelve more .on arriving at the third station it is found to have 248 passengers. Find the no of passengers in the beginning?
Let no of passengers in the beginning be x
After first station no passengers=(x-x/3)+280=2x/3 +280
After second station no passengers =1/2(2x/3+280)+12
½(2x/3+280)+12=248
2x/3+280=2*236
2x/3=192
x=288

28.if $a^2+b^2=177$and ab=54 then find the value of a+b/a-b?
$(a+b)^2=a^2+b^2+2ab=117+2*24=225$
a+b=15
$(a-b)^2=a^2+b^2-2ab=117-2*54$
a-b=3
a+b/a-b=15/3=5

29.find the value of (75983*75983- 45983*45983/30000)
Given expression=$(75983)^2-(45983)^2/(75983-45983)$
=$(a-b)^2/(a-b)$
=(a+b)(a-b)/(a-b)
=(a+b)
=75983+45983
=121966

30.find the value of $\left[\dfrac{\mathbf{343*343*343-113*113*113}}{\mathbf{343*343+343*113+113*113}}\right]$

Given expression= $\dfrac{(a^3-b^3)}{a^2+ab+b^2}$

=(a-b)

=(343-113)

.=230

31.Village X has a population of 68000,which is decreasing at the rate of 1200 per year.VillagyY has a population of 42000,which is increasing at the rate of 800 per year .in how many years will the population of the two villages be equal?

Let the population of two villages be equal after p years

Then,68000-1200p=42000+800p

2000p=26000

p=13

32.From a group of boys and girls,15 girls leave.There are then left 2 boys for each girl.After this,45 boys leave.There are then 5 girls for each boy.Find the number of girls in the beginning?

Let at present there be x boys.

Then,no of girls at present=5x

Before the boys had left:no of boys=x+45

And no of girls=5x

X+45=2*5x

9x=45

x=5

no of girls in the beginning=25+15=40

33.An employer pays Rs.20 for each day a worker works and for feits Rs.3 for each day is ideal at the end of sixty days a worker gets Rs.280 . for how many days did the worker remain ideal?

Suppose a worker remained ideal for x days then he worked for 60-x days

20*(60-x)-3x=280

1200-23x=280

23x=920

x=40

Ex 34.kiran had 85 currency notes in all , some of which were of Rs.100 denaomination and the remaining of Rs.50 denomination the total amount of all these currency note was Rs.5000.how much amount did she have in the denomination of Rs.50?

Let the no of fifty rupee notes be x

Then,no of 100 rupee notes =(85-x)

50x+100(85-x)=5000

x+2(85-x)=100
x=70
so,,required amount=Rs.(50*70)= Rs.3500

Ex. 35. When an amount was distributed among 14 boys, each of them got rs 80 more than the amount received by each boy when the same amount is distributed equally among 18 boys. What was the amount?

Sol. Let the total amount be Rs. X the,

$$\frac{x}{14} - \frac{x}{18} = 80 \Leftrightarrow \frac{2x}{126} = 80 \Leftrightarrow \frac{x}{63} = 63 \times 80 = 5040.$$

Hence the total amount is 5040.

Ex. 36. Mr. Bhaskar is on tour and he has Rs. 360 for his expenses. If he exceeds his tour by 4 days, he must cut down his daily expenses by Rs. 3. for how many days is Mr. Bhaskar on tour?

Sol. Suppose Mr. Bhaskar is on tour for x days. Then,

$$\frac{360}{x} - \frac{360}{x+4} = 3 \Leftrightarrow \frac{1}{x} - \frac{1}{x+4} = \frac{1}{120} \Leftrightarrow x(x+4) = 4 \times 120 = 480$$

$$\Leftrightarrow x^2 + 4x - 480 = 0 \Leftrightarrow (x+24)(x-20) = 0 \Leftrightarrow x = 20.$$

Hence Mr. Bhaskar is on tour for 20 days.

Ex. 37. Two pens and three pencils cost Rs 86. four Pens and a pencil cost Rs. 112. find the cost of a pen and that of a pencil.

Sol. Let the cost of a pen ands a pencil be Rs. X and Rs. Y respectively.

Then, 2x + 3y = 86 ….(i) and 4x + y =112.

Solving (i) and (ii), we get: x = 25 and y = 12.

Cost of a pen =Rs. 25 and the cost of a pencil =Rs. 12.

Ex. 38. Arjun and Sajal are friends . each has some money. If Arun gives Rs. 30 to Sajal, the Sajal will have twice the money left with Arjun. But, if Sajal gives Rs. 10 to Arjun, the Arjun will have thrice as much as is left with Sajal. How much money does each have?

Sol. Suppose Arun has Rs. X and Sjal has Rs. Y. then,

2(x-30)= y+30 => 2x-y =90 …(i)

and x +10 =3(y-10) => x-3y = - 40 …(ii)

Solving (i) and (ii), we get x =62 and y =34.

Arun has Rs. 62 and Sajal has Rs. 34.

Ex. 39. In a caravan, in addition to 50 hens there are 45 goats and 8 camels with some keepers. If the total number of feet be 224 more than the number of heads, find the number of keepers.

Sol. Let the number of keepers be x then,

Total number of heads $=(50 + 45 + 8 + x)= (103 + x)$.

Total number of feet $= (45 + 8) \times 4 + (50 + x) \times 2 =(312 +2x)$.

$$(312 + 2x)-(103 + x) =224 \Leftrightarrow x =15.$$

Hence, number of keepers =15.

SQUARE ROOTS AND CUBE ROOTS

IMPORTANT FACTS AND FORMULAE

Square Root: If $x^2 = y$, we say that the square root of y is x and we write, $\sqrt{y} = x$.
Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{196} = 14$.
Cube Root: The cube root of a given number x is the number whose cube is x. We denote the cube root of x by $\sqrt[3]{x}$.
Thus, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$, $\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$ etc.
Note:
1. $\sqrt{xy} = \sqrt{x} * \sqrt{y}$ 2. $\sqrt{(x/y)} = \sqrt{x} / \sqrt{y} = (\sqrt{x} / \sqrt{y}) * (\sqrt{y} / \sqrt{y}) = \sqrt{xy} / y$

SOLVED EXAMPLES

Ex. 1. Evaluate √6084 by factorization method .

Sol. **Method:** Express the given number as the product of prime factors. Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.
Thus, resolving 6084 into prime factors, we get:
$6084 = 2^2 \times 3^2 \times 13^2$
$\therefore \sqrt{6084} = (2 \times 3 \times 13) = 78.$

2	6084
2	3042
3	1521
3	507
13	169
	13

Ex. 2. Find the square root of 1471369.

Sol. **Explanation:** In the given number, mark off the digits in pairs starting from the unit's digit. Each pair and the remaining one digit is called a period.
Now, $1^2 = 1$. On subtracting, we get 0 as remainder.
Now, bring down the next period *i.e.,* 47.
Now, trial divisor is 1 x 2 = 2 and trial dividend is 47.
So, we take 22 as divisor and put 2 as quotient.
The remainder is 3.
Next, we bring down the next period which is 13.
Now, trial divisor is 12 x 2 = 24 and trial dividend is 313. So, we take 241 as dividend and 1 as quotient.
The remainder is 72.
Bring down the next period *i.e.,* 69.
Now, the trial divisor is 121 x 2 = 242 and the trial dividend is 7269. So, we take 3as quotient and 2423 as divisor. The remainder is then zero.
Hence, $\sqrt{1471369} = 1213$.

1	1471369 (1213
	1
22	47
	44
241	313
	241
2423	7269
	7269
	x

Ex. 3. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$.

Sol. Given expression = $\sqrt{248 + \sqrt{51 + 13}} = \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16.$

Ex. 4. If $a * b * c = \sqrt{(a + 2)(b + 3)} / (c + 1)$***, find the value of*** $6 * 15 * 3$.

Sol. $6 * 15 * 3 = \sqrt{(6 + 2)(15 + 3)} / (3 + 1) = \sqrt{8 * 18} / 4 = \sqrt{144} / 4 = 12 / 4 = 3.$

Ex. 5. ***Find the value of*** $\sqrt{25/16}$.

Sol. $\sqrt{25 / 16} = \sqrt{25} / \sqrt{16} = 5 / 4$

Ex. 6. ***What is the square root of 0.0009?***

Sol. $\sqrt{0.0009} = \sqrt{9 / 1000} = 3 / 100 = 0.03.$

Ex. 7. ***Evaluate*** $\sqrt{175.2976}$.

Sol. Method: We make even number of decimal places by affixing a zero, if necessary. Now, we mark off periods and extract the square root as shown.

$\therefore \sqrt{175.2976} = 13.24$

$$
\begin{array}{r|l}
1 & 175.2976\ (13.24 \\
 & 1 \\ \hline
23 & \ \ 75 \\
 & \ \ 69 \\ \hline
262 & \ \ \ 629 \\
 & \ \ \ 524 \\ \hline
2644 & \ \ \ 10576 \\
 & \ \ \ 10576 \\ \hline
 & \ \ \ \ \ \ \ x
\end{array}
$$

Ex. 8. ***What will come*** **in** ***place of question mark*** **in** ***each of the following*** **questions?**

(i) $\sqrt{32.4 / ?} = 2$ ***(ii)*** $\sqrt{86.49} + \sqrt{5 + (\ ?\ )^2} = 12.3.$

Sol. (i) Let $\sqrt{32.4 / x} = 2$. Then, $32.4/x = 4 \Leftrightarrow 4x = 32.4 \Leftrightarrow x = 8.1$.

(ii) Let $\sqrt{86.49} + \sqrt{5 + x^2} = 12.3$.

Then, $9.3 + \sqrt{5+x^2} = 12.3 \Leftrightarrow \sqrt{5+x^2} = 12.3 - 9.3 = 3$

$\Leftrightarrow 5 + x^2 = 9 \Leftrightarrow x^2 = 9 - 5 = 4 \Leftrightarrow x = \sqrt{4} = 2.$

Ex.9. Find the value of $\sqrt{0.289 / 0.00121}$.

Sol. $\sqrt{0.289 / 0.00121} = \sqrt{0.28900/0.00121} = \sqrt{28900/121} = 170 / 11$.

Ex.10. If $\sqrt{1 + (x / 144)} = 13 / 12$***, the find the value of x.***

Sol. $\sqrt{1 + (x / 144)} = 13 / 12 \Rightarrow (1 + (x / 144)) = (13 / 12)^2 = 169 / 144$

$\Rightarrow x / 144 = (169 / 144) - 1$

$\Rightarrow x / 144 = 25/144 \Rightarrow x = 25$.

Ex. 11. Find the value of √3 up to three places of decimal.

Sol.

1	3.000000	(1.732
	1	
27	200	
	189	
343	1100	
	1029	
3462	7100	
	6924	

$\therefore \sqrt{3} = 1.732$.

Ex. 12. ***If*** $\sqrt{3} = 1.732$***, find the value of*** $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$ ***correct to 3 places of decimal.*** **(S.S.C. 2004)**

Sol. $\sqrt{192} - (1 / 2)\sqrt{48} - \sqrt{75} = \sqrt{64 * 3} - (1/2) \sqrt{16 * 3} - \sqrt{25 * 3}$

$= 8\sqrt{3} - (1/2) * 4\sqrt{3} - 5\sqrt{3}$

$= 3\sqrt{3} - 2\sqrt{3} = \sqrt{3} = 1.732$

Ex. 13. Evaluate: $\sqrt{(9.5 * 0.0085 * 18.9) / (0.0017 * 1.9 * 0.021)}$

Sol. Given exp. = $\sqrt{(9.5 * 0.0085 * 18.9) / (0.0017 * 1.9 * 0.021)}$

Now, since the sum of decimal places in the numerator and denominator under the radical sign is the same, we remove the decimal.

$\therefore$ Given exp = $\sqrt{(95 * 85 * 18900) / (17 * 19 * 21)} = \sqrt{5 * 5 * 900} = 5 * 30 = 150$.

Ex. 14. ***Simplify***: $\sqrt{[(12.1)^2 - (8.1)^2] / [(0.25)^2 + (0.25)(19.95)]}$

Sol. Given exp. = $\sqrt{[(12.1 + 8.1)(12.1 - 8.1)] / [(0.25)(0.25 + 19.95)]}$

$= \sqrt{(20.2 * 4) /(0.25 * 20.2)} = \sqrt{4 / 0.25} = \sqrt{400 / 25} = \sqrt{16} = 4$.

Ex. 15. ***If*** $x = 1 + \sqrt{2}$ ***and*** $y = 1 - \sqrt{2}$***, find the value of*** $(x^2 + y^2)$.

Sol. $x^2 + y^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 2[(1)^2 + (\sqrt{2})^2] = 2 * 3 = 6.$

Ex. 16. ***Evaluate: √0.9 up to 3 places of decimal.***

Sol.

$$\begin{array}{r|l} 9 & 0.900000(0.948 \\ & \;\; 81 \\ \hline 184 & \;\;\;\; 900 \\ & \;\;\;\; 736 \\ \hline 1888 & \;\;\;\; 16400 \\ & \;\;\;\; 15104 \end{array}$$

$\therefore \sqrt{0.9} = 0.948$

Ex.17. If √15 = 3.88, find the value of √ (5/3).

Sol. $\sqrt{(5/3)} = \sqrt{(5 * 3) / (3 * 3)} = \sqrt{15} / 3 = 3.88 / 3 = 1.2933\ldots. = 1.29\overline{3}.$

Ex. 18. Find the least square number which is exactly divisible by 10,12,15 and 18.

Sol. L.C.M. of 10, 12, 15, 18 = 180. Now, $180 = 2 * 2 * 3 * 3 * 5 = 2^2 * 3^2 * 5$.

To make it a perfect square, it must be multiplied by 5.

$\therefore$ Required number $= (2^2 * 3^2 * 5^2) = 900$.

Ex. 19. Find the greatest number of five digits which is a perfect square.

(R.R.B. 1998)

Sol. Greatest number of 5 digits is 99999.

$$\begin{array}{r|l} 3 & 99999(316 \\ & 9 \\ \hline 61 & \;\; 99 \\ & \;\; 61 \\ \hline 626 & \;\; 3899 \\ & \;\; 3756 \\ \hline & \;\;\;\; 143 \end{array}$$

$\therefore$ Required number == (99999 - 143) = 99856.

Ex. 20. Find the smallest number that must be added to 1780 to make it a perfect square.

Sol.

$$\begin{array}{r|l} 4 & 1780\ (42 \\ & 16 \\ \hline 82 & \;\; 180 \\ & \;\; 164 \\ \hline & \;\;\;\; 16 \\ \hline \end{array}$$

$\therefore$ Number to be added $= (43)^2 - 1780 = 1849 - 1780 = 69$.

Ex. 21. √2 = 1.4142, find the value of √2 / (2 + √2).

Sol. $\sqrt{2} / (2 + \sqrt{2}) = \sqrt{2} / (2 + \sqrt{2}) * (2 - \sqrt{2}) / (2 - \sqrt{2}) = (2\sqrt{2} - 2) / (4 - 2)$

$= 2(\sqrt{2} - 1) / 2 = \sqrt{2} - 1 = 0.4142.$

22. If x = $(\sqrt{5} + \sqrt{3}) / (\sqrt{5} - \sqrt{3})$ and y = $(\sqrt{5} - \sqrt{3}) / (\sqrt{5} + \sqrt{3})$, find the value of $(x^2 + y^2)$.

Sol.

$x = [(\sqrt{5} + \sqrt{3}) / (\sqrt{5} - \sqrt{3})] * [(\sqrt{5} + \sqrt{3}) / (\sqrt{5} + \sqrt{3})] = (\sqrt{5} + \sqrt{3})^2 / (5 - 3)$

$= (5 + 3 + 2\sqrt{15}) / 2 = 4 + \sqrt{15}.$

$y = [(\sqrt{5} - \sqrt{3}) / (\sqrt{5} + \sqrt{3})] * [(\sqrt{5} - \sqrt{3}) / (\sqrt{5} - \sqrt{3})] = (\sqrt{5} - \sqrt{3})^2 / (5 - 3)$

$= (5 + 3 - 2\sqrt{15}) / 2 = 4 - \sqrt{15}.$

$\therefore \quad x^2 + y^2 = (4 + \sqrt{15})^2 + (4 - \sqrt{15})^2 = 2[(4)^2 + (\sqrt{15})^2] = 2 * 31 = 62.$

Ex. 23. Find the cube root of 2744.

Sol. **Method:** Resolve the given number as the product of prime factors and take the product of prime factors, choosing one out of three of the same prime factors. Resolving 2744 as the product of prime factors, we get:

2	2744
2	1372
2	686
7	343
7	49
	7

$2744 = 2^3 \times 7^3.$

$\therefore \quad \sqrt[3]{2744} = 2 \times 7 = 14.$

Ex. 24. By what least number 4320 be multiplied to obtain a number which is a perfect cube?

Sol. Clearly, $4320 = 2^3 * 3^3 * 2^2 * 5.$

To make it a perfect cube, it must be multiplied by $2 * 5^2$ i.e,50.

6.AVERAGE

Ex.1:Find the average of all prime numbers between 30 and 50?

Sol: there are five prime numbers between 30 and 50.

They are 31,37,41,43 and 47.

Therefore the required average=(31+37+41+43+47)/5 ⇔199/5 ⇔ 39.8.

Ex.2. find the average of first 40 natural numbers?

Sol: sum of first n natural numbers=n(n+1)/2;

So,sum of 40 natural numbers=(40*41)/2 ⇔820.

Therefore the required average=(820/40) ⇔20.5.

Ex.3. find the average of first 20 multiples of 7?

Sol: Required average =7(1+2+3+…….+20)/20 ⇔(7*20*21)/(20*2) ⇔(147/2)=73.5.

Ex.4. the average of four consecutive even numbers is 27. find the largest of these numbers?

Sol: let the numbers be x,x+2,x+4 andx+6. then,

(x+(x+2)+(x+4)+(x+6))/4) = 27

⇔(4x+12)/4 = 27

⇔x+3=27 ⇔ x=24.

Therefore the largest number=(x+6)=24+6=30.

Ex.5. there are two sections A and B of a class consisting of 36 and 44 students respectively. If the average weight of section A is 40kg and that of section B is 35kg, find the average weight of the whole class?

Sol: total weight of(36+44) students=(36*40+44*35)kg =2980kg.

Therefore weight of the total class=(2980/80)kg =37.25kg.

Ex:6.nine persons went to a hotel for taking their meals 8 of them spent Rs.12 each on their meals and the ninth spent Rs.8 more than the average expenditure of all the nine.What was the total money spent by them?

Sol: Let the average expenditure of all nine be Rs.x

Then 12*8+(x+8)=9x or 8x=104 or x=13.

Total money spent = 9x=Rs.(9*13)=Rs.117.

Ex.7: Of the three numbers, second is twice the first and is also thrice the third.If the average of the three numbers is 44.Find the largest number.

Sol: Let the third number be x.

Then second number = 3x.

First number=3x/2.

Therefore x+3x+(3x/2)=(44*3) or x=24

So largest number= 2^{nd} number=3x=72.

Ex.8:The average of 25 result is 18.The average of 1^{st} 12 of them is 14 & that of last 12 is 17.Find the 13^{th} result.
Sol: Clearly 13^{th} result=(sum of 25 results)-(sum of 24 results)
=(18*25)-(14*12)+(17*12)
=450-(168+204)
=450-372
=78.

Ex.9:The Average of 11 results is 16, if the average of the 1^{st} 6 results is 58 & that of the last 63. Find the 6^{th} result.
Sol: 6^{th} result = (58*6+63*6-60*11)=66

Ex.10:The average waight of A,B,C is 45 Kg. The avg wgt of A & B be 40Kg & that of B,C be 43Kg. Find the wgt of B.
Sol. Let A,B,c represent their individual wgts.
Then,
A+B+C=(45*3)Kg=135Kg
A+B=(40*2)Kg=80Kg & B+C=(43*2)Kg=86Kg
B=(A+B)+(B+C)-(A+B+C)
=(80+86-135)Kg
=31Kg.

Ex. 11. The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increases by3 months. Find the age of the teacher.
Sol. Total age of 39 persons = (39 x 15) years
= 585 years.
Average age of 40 persons= 15 yrs 3 months
= 61/4 years.
Total age of 40 persons = ((61/4)x 40) years= 610 years.

:. Age of the teacher = (610 - 585) years=25 years.

Ex. 12. The average weight of 10 oarsmen in a boat is increased by 1.8 kg when one of the crew, who weighs 53 kg is replaced by a new man. Find the weight of the new man.
Sol. Total weight increased =(1.8 x 10) kg =18 kg.
:. Weight of the new man =(53 + 18) kg =71 kg.

Ex. 13. There were 35 students in a hostel. Due to the admission of 7 new students, ;he expenses of the mess were increased by Rs. 42 per day while the average expenditure per head diminished by Rs 1. Wbat was the original expenditure of the mess?
Sol. Let the original average expenditure be Rs. x. Then,
42 (x - 1) - $35x$=42 ⇔ $7x$= 84 ⇔ x =12.
Original expenditure = Rs. (35 x 12) =Rs. 420. .

14. A batsman makes a score of 87 runs in the 17th inning and thus increases his avg by 3. Find his average after 17th inning.

Sol. Let the average after 17th inning = x.

Then, average after 16th inning = $(x - 3)$.

:. 16 (x - 3) + 87 = $17x$ or x = (87 - 48) = 39.

Ex.15. Distance between two stations A and B is 778 km. A train covers the journey from A to B at 84 km per hour and returns back to A with a uniform speed of 56 km perhour. Find the average speed of the train during the whole journey.

Sol. Required average speed = ((2xy)/(x+y)) km / hr

=(2 x 84 x 56)/(84+56)km/hr

= (2*84*56)/140 km/hr

=67.2 km/hr.

7. PROBLEMS ON NUMBERS

In this section, questions involving a set of numbers are put in the form of a puzzle. You have to analyze the given conditions, assume the unknown numbers and form equations accordingly, which on solving yield the unknown numbers.

SOLVED EXAMPLES

Ex.1. A number is as much greater than 36 as is less than 86. Find the number.

Sol. Let the number be x. Then, x - 36 = 86 - x => 2x = 86 + 36 = 122 => x = 61.
Hence, the required number is 61.

Ex. 2. Find a number such that when 15 is subtracted from 7 times the number, the Result is 10 more than twice the number. (Hotel Management, 2002)

Sol. Let the number be x. Then, 7x - 15 = 2x + 10 => 5x = 25 =>x = 5.
Hence, the required number is 5.

Ex. 3. The sum of a rational number and its reciprocal is 13/6. Find the number. (S.S.C. 2000)

Sol. Let the number be x.
Then, x + (1/x) = 13/6 => $(x^2 + 1)/x = 13/6$ => $6x^2 – 13x + 6 = 0$
=> $6x^2 – 9x – 4x + 6 = 0$ => (3x – 2) (2x – 3) = 0
⇨ x = 2/3 or x = 3/2
Hence the required number is 2/3 or 3/2.

Ex. 4. The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.

Sol. Let the numbers be x and (184 - *x).* Then,
(X/3) - ((184 – x)/7) = 8 => 7x – 3(184 – x) = 168 => 10x = 720 => x = 72.
So, the numbers are 72 and 112. Hence, smaller number = 72.

Ex. 5. The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers.

Sol. Let the number be x and y. Then,
x – y = 11 ----(i) and 1/5 *(x + y)* = 9 => *x* + *y* = 45 ----(ii)
Adding (i) and (ii), we get: 2x = 56 or x = 28. Putting x = 28 in (i), we get: y = 17.
Hence, the numbers are 28 and 17.

Ex. 6. If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers. (S.S.C. 2003)

Sol. Let the numbers be x and y. Then, x + y = 42 and xy = 437
x - *y* = sqrt[$(x + y)^2$ - 4xy] = sqrt[$(42)^2$ - 4 x 437] = sqrt[1764 – 1748] = sqrt[16] = 4.
Required difference = 4.

Ex. 7. The sum of two numbers is 16 and the sum of their squares is 113. Find the

numbers.

Sol. Let the numbers be x and (15 - *x).*

Then, $x^2 + (15 - x)^2 = 113$ => $x^2 + 225 + X^2 - 30x = 113$

=> $2x^2 - 30x + 112 = 0$ => $x^2 - 15x + 56 = 0$

=> $(x - 7)(x - 8) = 0$ => x = 7 or x = 8.

So, the numbers are 7 and 8.

Ex. 8. The average of four consecutive even numbers is 27. Find the largest of these numbers.

Sol. Let the four consecutive even numbers be x, x + 2, x + 4 and x + 6.

Then, sum of these numbers = (27 x 4) = 108.

So, $x + (x + 2) + (x + 4) + (x + 6) = 108$ or $4x = 96$ or $x = 24$.

:. Largest number = $(x + 6) = 30$.

Ex. 9. The sum of the squares of three consecutive odd numbers is 2531.Find the numbers.

Sol. Let the numbers be x, x + 2 and x + 4.

Then, $X^2 + (x + 2)^2 + (x + 4)^2 = 2531$ => $3x^2 + 12x - 2511 = 0$

=> $X^2 + 4x - 837 = 0$ => $(x - 27)(x + 31) = 0$ => x = 27.

Hence, the required numbers are 27, 29 and 31.

Ex. 10. Of two numbers, 4 times the smaller one is less then 3 times the 1arger one by 5. If the sum of the numbers is larger than 6 times their difference by 6, find the two numbers.

Sol. Let the numbers be x and *y,* such that x > y

Then, $3x - 4y = 5$ ...(i) and $(x + y) - 6(x - y) = 6$ => $-5x + 7y = 6$ …(ii)

Solving (i) and *(ii),* we get: x = 59 and *y* = 43.

Hence, the required numbers are 59 and 43.

Ex. 11. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1.If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number?

Sol. Let the ten's digit be x. Then, unit's digit = $(x + 3)$.

Sum of the digits = $x + (x + 3) = 2x + 3$. Number = $10x + (x + 3) = 11x + 3$.

11x+3 / 2x + 3 = 4 / 1 => $11x + 3 = 4(2x + 3)$ => $3x = 9$ => $x = 3$.

Hence, required number = $11x + 3 = 36$.

Ex. 12. A number consists of two digits. The sum of the digits is 9. If 63 is subtracted from the number, its digits are interchanged. Find the number.

Sol. Let the ten's digit be x. Then, unit's digit = (9 - *x).*

Number = $10x + (9 - x) = 9x + 9$.

Number obtained by reversing the digits = $10(9 - x) + x = 90 - 9x$.

therefore, $(9x + 9) - 63 = 90 - 9x$ => $18x = 144$ => x = 8.

So, ten's digit = 8 and unit's digit = 1.

Hence, the required number is 81.

Ex. 13. A fraction becomes 2/3 when 1 is added to both, its numerator and denominator.

And ,it becomes 1/2 when 1 is subtracted from both the numerator and denominator. Find

the fraction.

Sol. Let the required fraction be x/y. Then,

$x+1 / y+1 = 2 / 3 \Rightarrow 3x - 2y = -1$ …(i) and $x - 1 / y - 1 = 1 / 2$

$\Rightarrow 2x - y = 1$ …(ii)

Solving (i) and (ii), we get : $x = 3$, $y = 5$

therefore, Required fraction= 3 / 5.

Ex. 14. 50 is divided into two parts such that the sum of their reciprocals is 1/ 12.Find the two parts.

Sol. Let the two parts be x and (50 - *x*).

Then, $1 / x + 1 / (50 - x) = 1 / 12 \Rightarrow (50 - x + x) / x (50 - x) = 1 / 12$

$\Rightarrow x^2 - 50x + 600 = 0 \Rightarrow (x - 30) (x - 20) = 0 \Rightarrow x = 30$ or $x = 20$.

So, the parts are 30 and 20.

Ex. 15. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers)

Sol. Let the numbers be x, *y* and z. Then,

$x + y = 10$ ...*(i)* $y + z = 19$ ...*(ii)* $x + z = 21$ …(iii)

Adding (i) ,*(ii)* and *(iii),* we get: $2 (x + y + z) = 50$ or $(x + y + z) = 25$.

Thus, $x = (25 - 19) = 6$; $y = (25 - 21) = 4$; $z = (25 - 10) = 15$.

Hence, the required numbers are 6, 4 and 15.

8. PROBLEMS ON AGES

Ex. 1. Rajeev's age after 15 years will be 5 times his age 5 years back. What is the present age of Rajeev ? (Hotel Management,2002)

Sol. Let Rajeev's present age be x years. Then,

Rajeev's age after 15 years = (x + 15) years.

Rajeev's age 5 years back = (x - 5) years.

:. x + 15 = 5 (x - 5) ⇔ x + 15 = 5x - 25 ⇔ 4x = 40 ⇔ x = 10.

Hence, Rajeev's present age = 10 years.

Ex. 2. ***The ages*** **of** ***two persons differ by*** **16** ***years.*** **If 6** ***years ago, the*** **elder one be 3** ***times as old as the younger*** **one,** ***find their present ages.*** (A.A.O. Exam,2003)

Sol. Let the age of the younger person be x years.

Then, age of the elder person = (x + 16) years.

:. 3 (x - 6) = (x + 16 - 6) ⇔ 3x -18 = x + 10 ⇔ 2x = 28 ⇔ x = 14.

Hence, their present ages are 14 years and 30 years.

Ex. 3. ***The product*** **of** ***the ages*** **of** ***Ankit and Nikita is 240.*** **If** ***twice the*** **age of Nikita** ***is more than Ankit's age by*** **4** ***years, what is Nikita's age?*** (S.B.I.P.O,1999)

Sol. Let Ankit's age be x years. Then, Nikita's age = 240/xyears.

$\therefore$ 2 × (240 /x) – x = 4 ⇔ 480 – x^2 = 4x ⇔ x^2 + 4x – 480 = 0

⇔ (x+24)(x-20) = 0 ⇔ x = 20.

Hence, Nikita's age = (22_0) years = 12 years. 1

Ex. 4. ***The present age*** **of a** ***father is*** **3** ***years more than three times the*** **age of his** ***son. Three years hence, father's age will be 10 years more than twice the*** **age** ***of the*** **son.** ***Find the present age*** **of** ***the*** **father.** . **(S.S.C, 2003)**

Sol. Let the son's present age be x years. Then, father's present age = *(3x* + 3) years

$\therefore$ *(3x* + 3 + 3) = 2 (x + 3) + 10 ⇔ 3x + 6 = 2x + 16 ⇔ x = 10.

Hence, father's present age = *(3x* + 3) = ((3 × 10) + 3) years = 33 years.

Ex. 5. R*ohit was* **4** ***times as old as his son*** **8** ***years ago. After*** **8** ***years, Rohit will be twice as old as his son. What are their present ages?***

Sol. Let son's age 8 years ago be x years. Then, Rohit's age 8 years ago = 4x years.

Son's age after 8 years = (x + 8) + 8 = (x + 16) years.

Rohit's age after 8 years = *(4x* + 8) + 8 = *(4x*+ 16) years.

$\therefore$ 2 (x + 16) = 4x + 16 ⇔ 2x = 16 ⇔ x = 8.

Hence, son's 'present age = (x + 8) = 16 years.

Rohit's present age = *(4x* + 8) = 40 years.

Ex. 6. One *year ago, the ratio* of *Gaurav's and Sachin's age was* 6: 7 *respectively. Four years hence, this ratio would become* 7: 8. *How old is Sa chin* ?

(NABARD, 2002)

Sol:

. Let Gaurav's and Sachin's ages one year ago be 6x and 7x years respectively. Then, Gaurav's age 4 years hence = $(6x + 1) + 4 = (6x + 5)$ years.

Sachin's age 4 years hence = $(7x + 1) + 4 = (7x + 5)$ years.

$$\frac{6x+5}{7x+5} = \frac{7}{8} \Leftrightarrow 8(6x+5) = 7(7x + 5) \Leftrightarrow 48x + 40 = 49x + 35 \Leftrightarrow x = 5.$$

Hence, Sachin's present age = $(7x + 1) = 36$ years.

,7. *Abhay's age after six years will be three-seventh* of *his fathers age. Ten years* ago the *ratio* of *their ages was 1* : 5. *What is Abhay's father's age* at *present?*

Sol. Let the ages of Abhay and his father 10 years ago be x and 5x years respectively. Then,

Abhay's age after 6 years = $(x + 10) + 6 = (x + 16)$ years.

Father's age after 6 years = $(5x + 10) + 6 = (5x + 16)$ years.

$$\backslash x + 16) = \frac{3}{7}(5x + 16) \Leftrightarrow 7(x + 16) = 3(5x + 16) \Leftrightarrow 7x + 112 = 15x + 48$$

$$\Leftrightarrow 8x = 64 \Leftrightarrow x = 8.$$

Hence, Abhay's father's present age = $(5x + 10) = 50$ years.

9. SURDS AND INDICES

I IMPORTANT FACTS AND FORMULAE I

1. LAWS OF INDICES:

(i) $a^m \times a^n = a^{m+n}$
(ii) $a^m / a^n = a^{m-n}$
(iii) $(a^m)^n = a^{mn}$
(iv) $(ab)^n = a^n b^n$
(v) $(a/b)^n = (a^n / b^n)$
(vi) $a^0 = 1$

2. SURDS: *Let a be a rational number and* n *be a positive integer such that* $a^{1/n} = {}^n sqrt(a)$ *is irrational. Then* ^{n}sqrt(a) is *called a surd of order* n.

3. LAWS OF SURDS:

(i) $^n\sqrt{a} = a^{1/2}$
(ii) $^n\sqrt{ab} = {}^n\sqrt{a} * {}^n\sqrt{b}$
(iii) $^n\sqrt{a/b} = {}^n\sqrt{a} \ / \ {}^n\sqrt{b}$
(iv) $({}^n\sqrt{a})^n = a$
(v) $^m\sqrt{({}^n\sqrt{(a)})} = {}^{mn}\sqrt{(a)}$
(vi) $({}^n\sqrt{a})^m = {}^n\sqrt{a^m}$

I SOLVED EXAMPLES

Ex. 1. Simplify : (i) $(27)^{2/3}$ (ii) $(1024)^{-4/5}$ (iii) $(8 / 125)^{-4/3}$

Sol . (i) $(27)^{2/3} = (3^3)^{2/3} = 3^{(3 * (2/3))} = 3^2 = 9$
(ii) $(1024)^{-4/5} = (4^5)^{-4/5} = 4^{\{5 * ((-4)/5)\}} = 4^{-4} = 1 / 4^4 = 1 / 256$
(iii) $(8 / 125)^{-4/3} = \{(2/5)^3\}^{-4/3} = (2/5)^{\{3 * (-4/3)\}} = (2 / 5)^{-4} = (5 / 2)^4 = 5^4 / 2^4 = 625 / 16$

Ex. 2. Evaluate: (i) $(.00032)^{3/5}$ *(ii)l* $(256)^{0.16} \times (16)^{0.18}$.

Sol. (i) $(0.00032)^{3/5} = (32 / 100000)^{3/5}. = (2^5 / 10^5)^{3/5} = \{(2 / 10)^5\}^{3/5} = (1 / 5)^{(5 * 3 / 5)} = (1/5)^3 = 1 / 125$
(ii) $(256)^{0.16} * (16)^{0.18} = \{(16)^2\}^{0.16} * (16)^{0.18} = (16)^{(2 * 0.16)} * (16)^{0.18}$
$=(16)^{0.32} * (16)^{0.18} = (16)^{(0.32+0.18)} = (16)^{0.5} = (16)^{1/2} = 4.$

Ex. 3. *What is the quotient when* $(x^{-1} - 1)$ *is divided by* $(x - 1)$?

Sol. $\dfrac{x^{-1} - 1}{x - 1} = \dfrac{(1/x) - 1}{x - 1} = \dfrac{1 - x}{x} * \dfrac{1}{(x - 1)} = \dfrac{-1}{x}$

Hence, the required quotient is $-1/x$

Ex. 4. If $2^{x - 1} + 2^{x + 1} = 1280$*, then find the value* of x.

Sol. $2^{x - 1} + 2^{X + 1} = 1280 \Leftrightarrow 2^{x-1}(1 + 2^2) = 1280$

$$\Leftrightarrow 2^{x-1} = 1280 / 5 = 256 = 2^8 \Leftrightarrow x - 1 = 8 \Leftrightarrow x = 9.$$

Hence, x = 9.

Ex. 5. *Find* the value of $[5(8^{1/3} + 27^{1/3})^3]^{1/4}$

Sol. $[5(8^{1/3} + 27^{1/3})^3]^{1/4} = [5\{(2^3)^{1/3} + (3^3)^{1/3}\}^3]^{1/4} = [5\{(2^{3 * 1/3})^{1/3} + (3^{3 * 1/3})^{1/3}\}^3]^{1/4}$

$$= \{5(2+3)^3\}^{1/4} = (5 * 5^3)^{1/4} = 5^{(4 * 1/4)} = 5^1 = 5.$$

Ex. 6. Find the Value of $\{(16)^{3/2} + (16)^{-3/2}\}$

Sol. $[(16)^{3/2} + (16)^{-3/2} = (4^2)^{3/2} + (4^2)^{-3/2} = 4^{(2 * 3/2)} + 4^{\{2 * (-3/2)\}}$

$$= 4^3 + 4^{-3} = 4^3 + (1/4^3) = (64 + (1/64)) = 4097/64.$$

Ex. 7. *If* $(1/5)^{3y} = 0.008$*, then find the value of* $(0.25)^y$.

Sol. $(1/5)^{3y} = 0.008 = 8/1000 = 1/125 = (1/5)^3 \Leftrightarrow 3y = 3 \Leftrightarrow Y = 1.$

$\therefore (0.25)^y = (0.25)^1 = 0.25.$

Ex. 8. *Find the value* of $\dfrac{(243)^{n/5} \times 3^{2n+1}}{9^n \times 3^{n-1}}$.

Sol. $\dfrac{(243)^{n/5} x 3^{2n+l}}{(3^2)^n \times 3^{n-1}} = \dfrac{3^{(5 * n/5)} \times 3^{2n+l}}{3^{2n} \times 3^{n-1}} = \dfrac{3^n \times 3^{2n+1}}{32n \times 3^{n-l}}$

$$= \frac{3^{n+(2n+1)}}{3^{2n+n-1}} = \frac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+l)-(3n-l)} = 3^2 = 9.$$

Ex. 9. *Find the value Of* $(2^{1/4}-1)(2^{3/4}+2^{1/2}+2^{1/4}+1)$

Sol.

Putting $2^{1/4} = x$, we get :

$(2^{1/4}-1)(2^{3/4}+2^{1/2}+2^{1/4}+1)=(x-1)(x^3+x^2+x+1)$, *where* $x = 2^{1/4}$

$=(x-1)[x^2(x+1)+(x+1)]$

$=(x-1)(x+1)(x^2+1) = (x^2-1)(x^2+1)$

$=(x^4-1) = [(2^{1/4})^4-1] = [2^{(1/4*4)} - 1] = (2-1) = 1.$

Ex. 10. Find the value of $\dfrac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$

Sol. $\dfrac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \dfrac{6^{2/3} \times (6^7)^{1/3}}{(6^6)^{1/3}} = \dfrac{6^{2/3} \times 6^{(7 * 1/3)}}{6^{(6 * 1/3)}} = \dfrac{6^{2/3} \times 6^{(7/3)}}{6^2}$

$$=6^{2/3} \times 6^{((7/3)-2)} = 6^{2/3} \times 6^{1/3} = 6^1 = 6.$$

Ex. 11. If $x= y^a$, $y=z^b$ *and* $z=x^c$,*then find the value of abc.*

Sol. $z^1 = x^c = (y^a)^c$ [*since* $x= y^a$]

$=y^{(ac)} = (z^b)^{ac}$ [*since* $y=z^b$]

$=z^{b(ac)} = z^{abc}$

$\therefore$ $abc = 1.$

Ex. 12. Simplify $[(x^a / x^b)^\wedge(a^2+b^2+ab)] * [(x^b / x^c)^\wedge b^2+c^2+bc)] * [(x^c/x^a)^\wedge(c^2+a^2+ca)]$

Sol.

Given Expression

$= [\{x^{(o-b)}\}^\wedge(a^2 + b^2 + ob)].['(x^{(b-c)}\}^\wedge (b^2 + c^2 + bc)].['(x^{(c-a)}\}^\wedge(c^2 + a^2 + ca])$

$= [x^{(a-b)(a2+b2+ab)} . x^{(b-c)(b2+c2+bc)} . x^{(c-a)(c2+a2+ca)}]$

$= [x^\wedge(a^3-b^3)].[x^\wedge(b^3-e^3)].[x^\wedge(c^3-a^3)] = x^\wedge(a^3-b^3+b^3-c^3+c^3-a^3) = x^0 = 1.$

Ex. 13. *Which is larger* $\sqrt{2}$ or $\sqrt[3]{3}$?

Sol. Given surds are of order 2 and 3. Their L.C.M. is 6. Changing each to a surd of order 6, we get:

$$\sqrt{2} = 2^{1/2} = 2^{((1/2)*(3/2))} = 2^{3/6} = 8^{1/6} = \sqrt[6]{8}$$
$$\sqrt[3]{3} = 3^{1/3} = 3^{((1/3)*(2/2))} = 3^{2/6} = (3^2)^{1/6} = (9)^{1/6} = \sqrt[6]{9}.$$

Clearly, $\sqrt[6]{9} > \sqrt[6]{8}$ and hence $\sqrt[3]{3} > \sqrt{2}$.

Ex. 14. *Find the largest from among* $4\sqrt{6}$, $\sqrt{2}$ *and* $\sqrt[3]{4}$.

Sol. Given surds are of order 4, 2 and 3 respectively. Their L.C,M, is 12, Changing each to a surd of order 12, we get:

$\sqrt[4]{6} = 6^{1/4} = 6^{((1/4)*(3/3))} = 6^{3/12} = (6^3)^{1/12} = (216)^{1/12.}$
$\sqrt{2} = 2^{1/2} = 2^{((1/2)*(6/6))} = 2^{6/12} = (2^6)^{1/12} = (64)^{1/12}$.
$\sqrt[3]{4} = 4^{1/3} = 4^{((1/3)*(4/4))} = 4^{4/12} = (4^4)^{1/12} = (256)^{1/12}$.

Clearly, $(256)^{1/12} > (216)^{1/12} > (64)^{1/12}$

Largest one is $(256)^{1/12}$. i.e. $\sqrt[3]{4}$.

10.PERCENTAGE

IMPORTANT FACTS AND FORMULAE

1. **Concept of Percentage :** By a certain percent ,we mean that many hundredths. Thus x percent means x hundredths, written as x%.

   To express x% as a fraction : We have , x% = x/100.

   Thus, 20% =20/100 =1/5; 48% =48/100 =12/25, etc.

   To express a/b as a percent : We have, a/b =((a/b)*100)%.

   Thus, ¼ =[(1/4)*100] = 25%; 0.6 =6/10 =3/5 =[(3/5)*100]% =60%.

2. If the price of a commodity increases by R%, then the reduction in consumption so asnot to increase the expenditure is

   [R/(100+R))*100]%.

   If the price of the commodity decreases by R%,then the increase in consumption so as to decrease the expenditure is

   [(R/(100-R)*100]%.

3. Results on Population : Let the population of the town be P now and suppose it increases at the rate of

R% per annum, then :

1. Population after nyeras = P [1+(R/100)]^n.
2. Population n years ago = P /[1+(R/100)]^n.

4. Results on Depreciation : Let the present value of a machine be P. Suppose it depreciates at the rate

R% per annum. Then,

1. Value of the machine after n years = $P[1-(R/100)]^n$.
2. Value of the machine n years ago = $P/[1-(R/100)]^n$.

5. If A is R% more than B, then B is less than A by

[(R/(100+R))*100]%.

If A is R% less than B , then B is more than A by

[(R/(100-R))*100]%.

SOLVED EXAMPLES

Ex. 1. Express each of the following as a fraction :

(i) 56% **(ii) 4%** **(iii) 0.6%** **(iv) 0.008%**

sol. (i) 56% = 56/100= 14/25. (ii) 4% =4/100 =1/25.
(iii) 0.6 =6/1000 = 3/500. (iv) 0.008 = 8/100 = 1/1250.

Ex. 2. Express each of the following as a Decimal :

(i) 6% **(ii)28%** **(iii) 0.2%** **(iv) 0.04%**

Sol. (i) 6% = 6/100 =0.06. (ii) 28% = 28/100 =0.28.
(iii) 0.2% =0.2/100 = 0.002. (iv) 0.04%= 0.04/100 =0.004.

Ex. 3. Express each of the following as rate percent :

(i) 23/36 **(ii) 6 ¾** **(iii) 0.004**

Sol. (i) 23/36 = [(23/36)*100]% = [575/9]% = 63 8/9%.

(ii) 0.004 = [(4/1000)*100]% = 0.4%.

(iii) 6 ¾ =27/4 =[(27/4)*100]% = 675%.

Ex. 4. Evaluate :

(i) 28% of 450+ 45% of 280

(ii) 16 2/3% of 600 gm- 33 1/3% of 180 gm

Sol. (i) 28% of 450 + 45% of 280 =[(28/100)*450 + (45/100)*280] = (126+126) =252.

(iii) 16 2/3% of 600 gm –33 1/3% of 180 gm = [((50/3)*(1/100)*600) – ((100/3)*(1/3)*280)]gm = (100-60) gm = 40gm.

Ex. 5.
(i) 2 is what percent of 50 ?
(ii) ½ is what percent of 1/3 ?
(iii)What percent of 8 is 64 ?
(iv)What percent of 2 metric tones is 40 quintals ?
(v)What percent of 6.5 litres is 130 ml?

Sol.

(i) Required Percentage = [(2/50)*100]% = 4%.

(ii) Required Percentage = [(1/2)*(3/1)*100]% = 150%.

(iii)Required Percentage = [(84/7)*100]% = 1200%.

(iv) 1 metric tonne = 10 quintals.

Required percentage = [(40/(2 * 10)) * 100]% = 200%.

(v) Required Percentage = [(130/(6.5 * 1000)) * 100]% = 2%.

Ex. 6.
Find the missing figures :

(i) ?% of 25 = 20125 (ii) 9% of ? = 63 (iii) 0.25% of ? = 0.04

Sol.

(i) Let x% of 25 = 2.125. Then , (x/100)*25 = 2.125
X = (2.125 * 4) = 8.5.

(ii) Let 9% of x =6.3. Then , 9*x/100 = 6.3
X = [(6.3*100)/9] =70.

(iii) Let 0.25% of x = 0.04. Then , 0.25*x/100 = 0.04
X= [(0.04*100)/0.25] = 16.

Ex. 7.
Which is greatest in 16 (2/3) %, 2/5 and 0.17 ?

Sol. 16 (2/3)% =[(50/3)*)1/100)] = 1/6 = 0.166, 2/15 = 0.133. Clearly, 0.17 is the greatest.

Ex. 8.
If the sales tax reduced from 3 1/2 % to 3 1/3%, then what difference does it make to a person who purchases an article with market price of Rs. 8400 ?

Sol. Required difference = [3 ½ % of Rs.8400] – [3 1/3 % of Rs.8400]

= [(7/20-(10/3)]% of Rs.8400 =1/6 % of Rs.8400

= Rs. [(1/6)8(1/100)*8400] = Rs. 14.

Ex. 9. An inspector rejects 0.08% of the meters as defective. How many will be examine to project ?

Sol. Let the number of meters to be examined be x.

Then, 0.08% of x =2

[(8/100)*(1/100)*x] = 2

x = [(2*100*100)/8] = 2500.

Ex. 10. Sixty five percent of a number is 21 less than four fifth of that number. What is the number ?

Sol. Let the number be x.

Then, 4*x/5 –(65% of x) = 21

4x/5 –65x/100 = 21

5 x = 2100

x = 140.

Ex.11. Difference of two numbers is 1660. If 7.5% of the number is 12.5% of the other number , find the number ?

Sol. Let the numbers be x and y. Then , 7.5 % of x =12.5% of y

X = 125*y/75 = 5*y/3.

Now, x-y =1660

5*y/3 –y =1660

2*y/3= 1660

y =[(1660*3)/2] =2490.

One number = 2490, Second number =5*y/3 =4150.

Ex. 12.

In expressing a length 810472 km as nearly as possible with three significant digits , find the percentage error.

Sol. Error = (81.5 – 81.472)km = 0.028.

Required percentage = [(0.028/81.472)*100]% = 0.034%.

Ex. 13.

In an election between two candidates, 75% of the voters cast thier thier votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of the total valid votes. Find the total number of votes enrolled in that election.

Sol.

Let the number of votes enrolled be x. Then ,

Number of votes cast =75% of x. Valid votes = 98% of (75% of x).

75% of (98% of (75%of x)) =9261.

[(75/100)*(98/100)*(75/100)*x] =9261.

X = [(9261*100*100*100)/(75*98*75)] =16800.

Ex.14. Shobha's mathematics test had 75 problems i.e.10 arithmetic, 30 algebra and 35 geometry problems. Although she answered 70% of the arithmetic ,40% of the algebra, and 60% of the geometry problems correctly. she did not pass the test because she got less than 60% of the problems right. How many more questions she would have to answer correctly to earn 60% of the passing grade?

Sol. Number of questions attempted correctly=(70% of 10 + 40% of 30 + 60% 0f 35)
=7 + 12+21= 45

questions to be answered correctly for 60% grade=60% of 75 = 45 therefore required number of questions= (45-40) = 5.

Ex.15. if 50% of (x-y) = 30% of (x+y) then what percent of x is y?

Sol.50% of (x-y)=30% of(x+y) ⇔ (50/100)(x-y)=(30/100)(x+y)
⇔5(x-y)=3(x+y) ⇔ 2x=8y ⇔ x=4y
therefore required percentage =((y/x) X 100)% = ((y/4y) X 100) =25%

Ex.16. Mr.Jones gave 40% of the money he had to his wife. he also gave 20% of the remaining amount to his 3 sons. half of the amount now left was spent on miscellaneous items and the remaining amount of Rs.12000 was deposited in the bank. how much money did Mr.jones have initially?

Sol. Let the initial amount with Mr.jones be Rs.x then,
Money given to wife= Rs.(40/100)x=Rs.2x/5.Balance=Rs(x-(2x/5)=Rs.3x/5.
Money given to 3 sons= Rs(3X((20/200) X (3x/5)) = Rs.9x/5.
Balance = Rs.((3x/5) – (9x/25))=Rs.6x/25.
Amount deposited in bank= Rs(1/2 X 6x/25)=Rs.3x/25.

Therefore 3x/25=12000 ⇔ x= ((12000 x 35)/3)=100000

So Mr.Jones initially had Rs.1,00,000 with him.

Short-cut Method : Let the initial amount with Mr.Jones be Rs.x

Then,(1/2)[100-(3*20)]% of x=12000

⇔ (1/2)*(40/100)*(60/100)*x=12000
⇔x=((12000*25)/3)=100000

Ex 17 10% of the inhabitants of village having died of cholera.,a panic set in , during which 25% of the remaining inhabitants left the village. The population is then reduced to 4050. Find the number of original inhabitants.
Sol:
Let the total number of orginal inhabitants be x.
((75/100))*(90/100)*x)=4050 ⇔ (27/40)*x=4050

⇔x=((4050*40)/27)=6000.

Ex.18 A salesman`s commission is 5% on all sales upto Rs.10,000 and 4% on all sales exceeding this.He remits Rs.31,100 to his parent company after deducing his commission . Find the total sales.
Sol:
Let his total sales be Rs.x.Now(Total sales) – (Commission)=Rs.31,100
x-[(5% of 10000 + 4% of (x-10000)]=31,100
x-[((5/100)*10000 + (4/100)*(x-10000)]=31,100

⇔x-500-((x-10000)/25)=31,100
⇔x-(x/25)=31200 ⇔ 24x/25=31200⇔x=[(31200*25)/24)=32,500.
Total sales=Rs.32,500

Ex .19 Raman`s salary was decreased by 50% and subsequently increased by 50%.How much percent does he lose?
Sol:
Let the original salary = Rs.100
New final salary=150% of (50% of Rs.100)=
Rs.((150/100)*(50/100)*100)=Rs.75.
Decrease = 25%

Ex.20 Paulson spends 75% of his income. His income is increased by 20% and he increased his expenditure by 10%.Find the percentage increase in his savings .
Sol:
Let the original income=Rs.100 . Then , expenditure=Rs.75 and savings =Rs.25
New income =Rs.120 , New expenditure =
Rs.((110/100)*75)=Rs.165/2
New savings = Rs.(120-(165/2)) = Rs.75/2
Increase in savings = Rs.((75/2)-25)=Rs.25/2
Increase %= ((25/2)*(1/25)*100)% = 50%.

Ex21. The salary of a person was reduced by 10% .By what percent should his reduced salary be raised so as to bring it at par with his original salary ?
Sol:
Let the original salary be Rs.100 . New salary = Rs.90.
Increase on 90=10 , Increase on 100=((10/90)*100)%
= (100/9)%

Ex.22 When the price fo a product was decreased by 10% , the number sold increased by 30%. What was the effect on the total revenue ?
Sol:
Let the price of the product be Rs.100 and let original sale be 100 pieces.
Then , Total Revenue = Rs.(100*100)=Rs.10000.
New revenue = Rs.(90*130)=Rs.11700.
Increase in revenue = ((1700/10000)*100)%=17%.

Ex 23 . If the numerator of a fraction be increased by 15% and its denominator be diminished by 8% , the value of the fraction is 15/16. Find the original fraction.
Sol:
Let the original fraction be x/y.
Then (115%of x)/(92% of y)=15/16 => (115x/92y)=15/16
⇨ ((15/16)*(92/115))=3/4

Ex.24 In the new budget , the price of kerosene oil rose by 25%. By how much percent must a person reduce his consumption so that his expenditure on it does not increase ?

Sol:

Reduction in consumption = [((R/(100+R))*100]%

⇨ [(25/125)*100]%=20%.

Ex.25 The population of a town is 1,76,400 . If it increases at the rate of 5% per annum , what will be its population 2 years hence ? What was it 2 years ago ?

Sol:

Population after 2 years = 176400*[1+(5/100)]^2

=[176400*(21/20)*(21/40)]

= 194481.

Population 2 years ago = 176400/[1+(5/100)]^2

=[716400*(20/21)*(20/21)]= 160000.

Ex.26 The value of a machine depreiates at the rate of 10% per annum. If its present is Rs.1,62,000 what will be its worth after 2 years ? What was the value of the machine 2 years ago ?

Sol.

Value of the machine after 2 years

=Rs.[162000*(1-(10/100))^2] = Rs.[162000*(9/10)*(9/10)]

=Rs. 131220

Value of the machine 2 years ago

= Rs.[162000/(1-(10/100)^2)]=Rs.[162000*(10/9)*(10/9)]=Rs.200000

Ex27. During one year, the population of town increased by 5% . If the total population is 9975 at the end of the second year , then what was the population size in the beginning of the first year ?

Sol:

Population in the beginning of the first year

= 9975/[1+(5/100)]*[1-(5/100)] = [9975*(20/21)*(20/19)]=10000.

Ex.28 If A earns 99/3% more than B,how much percent does B earn less then A ?

Sol:

Required Percentage = [((100/3)*100)/[100+(100/3)]]%

=[(100/400)*100]%=25%

Ex. 29 If A`s salary is 20% less then B`s salary , by how much percent is B`s salary more than A`s ?

Sol:

Required percentage = [(20*100)/(100-20)]%=25%.

Ex30 .How many kg of pure salt must be added to 30kg of 2% solution of salt and water to increase it to 10% solution ?
Sol:
Amount of salt in 30kg solution = [(20/100)*30]kg=0.6kg
Let x kg of pure salt be added
Then , (0.6+x)/(30+x)=10/100⇔60+100x=300+10x
⇔90x=240 ⇔ x=8/3.

Ex 31. Due to reduction of 25/4% in the price of sugar , a man is able to buy 1kg more for Rs.120. Find the original and reduced rate of sugar.
Sol:
Let the original rate be Rs.x per kg.
Reduced rate = Rs.[(100-(25/4))*(1/100)*x}]=Rs.15x/16per kg
120/(15x/16)-(120/x)=1 ⇔ (128/x)-(120/x)=1
⇔ x=8.
So, the original rate = Rs.8 per kg
Reduce rate = Rs.[(15/16)*8]per kg = Rs.7.50 per kg

Ex.32 In an examination , 35% of total students failed in Hindi , 45% failed in English and 20% in both . Find the percentage of those who passed in both subjects .
Sol:
Let A and B be the sets of students who failed in Hindi and English respectively .
Then , n(A) = 35 , n(B)=45 , n(A∩B)=20.
So , n(A∪B)=n(A)+n(B)- n(A∩B)=35+45-20=60.
Percentage failed in Hindi and English or both=60%
Hence , percentage passed = (100-60)%=40%

Ex33. In an examination , 80% of the students passed in English , 85% in Mathematics and 75% in both English and Mathematics. If 40 students failed in both the subjects , find the total number of students.
Sol:
Let the total number of students be x .
Let A and B represent the sets of students who passed in English and Mathematics respectively .
Then , number of students passed in one or both the subjects
= n(A∪B)=n(A)+n(B)- n(A∩B)=80% of x + 85% of x –75% of x
=[(80/100)x+(85/100)x-(75/100)x]=(90/100)x=(9/10)x
Students who failed in both the subjects = [x-(9x/10)]=x/10.
So, x/10=40 of x=400 .
Hence ,total number of students = 400.

11. PROFIT AND LOSS

IMPORTANT FACTS

COST PRICE: THE PRICE AT WHICH ARTICLE IS PURCHASED.ABBREVATED AS C.P.

SELLING PRICE: THE PRICE AT WHICH ARTICLE IS SOLD.

PROFIT OR GAIN:IF SP IS GREATER THAN CP,THE SELLING PRICE IS SAID TO HAVE PROFIT OR GAIN.

LOSS: IF SPIS LESS THAN CP,THE SELLER IS SAID TO INCURED A LOSS.

FORMULA

1.GAIN=(SP)-(CP). 2.LOSS=(CP)-(SP).
3.LOSS OR GAIN IS ALWAYS RECKONED ON CP
4. GAIN %={GAIN*100}/CP.
5.LOSS%={LOSS*100}/CP.
6.SP={(100+GAIN%) /100}*CP.
7.SP={(100-LOSS%)/100}*CP.
8.{100/(100+GAIN%)} *SP
9.CP=100/(100-LOSS%)}*SP
10.IF THE ARTICLE IS SOLD AT A GAIN OF SAY 35%, THEN SP =135% OF CP
11.IF A ARTICLE IS SOLD AT A LOSS OF SAY 35%. THEN SP=65% OF CP.
12.WHEN A PERSON SELLS TWO ITEMS,ONE AT A GAIN OF X% AND OTHER AT A LOSS OF X%.THEN THE SELLER ALWAYS INCURES A LOSS GIVEN:
{LOSS%=(COMON LOSS AND GAIN)^2}/10.=(X/10)^2
13.IF THE TRADER PROFESSES TO SELL HIS GOODS AT CP BUT USES FALSE WEIGHTS,THEN
GAIN=[ERROR/(TRUE VALUE)-(ERROR)*100]%

SOLVED PROBLEMS

ex.1 A man buys an article for rs.27.50 and sells it for rs.28.50. find his gain %.

sol. cp=rs27.50, sp=rs 28.50
gain=rs(28.50 –27.50)=rs1.10
so gain%={(1.10/27.50)*100}=4%

Ex.2. If the a radio is sold for rs 490 and sold for rs 465.50.find loss%.

sol. cp=rs490,sp= 465.50.
loss=rs(490-465.50)=rs 24.50.
loss%=[(24.50/490)*100]%=5%

Ex.3.find S.P when

(i)CP=56.25,gain=20%.
sol.
(i)SP =20% of rs 56.25 ,=rs{(120/100)*56.25}=rs67.50.

(ii)CP=rs 80.40,loss=5%
sol: sp=85% of rs 80.40
=rs {(85/100)*80.40}=rs 68.34.

ex.4 find cp when:

(i) sp =rs 40.60 : gain=16%
(ii) sp=rs51.70:loss=12%

(i) cp=rs{(100/116)*40.60}=rs 35.
(ii) cp=rs{(100/88)*51.87}=rs58.75.

ex.5 A person incures loss for by selling a watch for rs1140.at what price should the watch be sold to earn a 5% profit ?

sol. let the new sp be rsx.then

(100-loss%) : (1st sp)=(100+gain%) (2nd sp)

⇨ {(100-5)/1400}={(100+5)/x}=> x={(105*1140)/95} =1260.

⇨

ex.6 A book was sold for rs 27.50 with a profit of 10%. if it were sold for rs25.75, then what would be % of profit or loss?

sol. SP=rs 27.50: profit =10%.
sol. CP=rs {(100/110)*27.50}=rs 25.
When sp =Rs25.75 ,profit =Rs(25.75-25)=Rs 0.75

Profit%={(0.75/25)*100}%=25/6%=3%

Ex7 .If the cost price is 96% of sp then whqt is the profit %

Sol. sp=Rs100 : then cp=Rs 96:profit =Rs 4.

Profit={(4/96)*100}%=4.17%

Ex.8. The cp of 21 articles is equal to sp of 18 articles.find gain or loss %

CP of each article be Rs 1

CP of 18 articles =Rs18 ,sp of 18 articles =Rs 21.

Gain%=[(3/18)*100]%=50/3%

Ex.9 By selling 33 metres of cloth , one gains the selling price of 11 metres . Find the gain percent .

Sol:

(SP of 33m)-(CP of 33m)=Gain=SP of 11m

SP of 22m = CP of 33m

Let CP of each metre be Re.1 , Then, CP of 22m= Rs.22,SP of 22m=Rs.33.

Gain%=[(11/22)*100]%=50%

Ex10 A vendor bought bananas at 6 for Rs.10 and sold them at Rs.4 for Rs.6 .Find his gain or loss percent .

Sol:

Suppose , number of bananas bought = LCM of 6 and 4=12

CP=Rs.[(10/6)*12]=Rs.20 ; SP= Rs[(6/4)*12]=Rs.18

Loss%=[(2/20)*100]%=10%

Ex.11. A man brought toffees at for a rupee. How many for a rupee must he sell to gain 50%?

Sol. C.P of 3 toffees=Re 1; S.P of 3 toffees =150% of Re.1=3/2.

For Rs.3/2, toffees sold =3, for Re.1, toffees sold = [3*(2/3)] = 2.

Ex. 12.A grocer purchased 80 kg of sugar at Rs.13.50 per kg and mixed it with 120kg sugar at Rs.16per kg. At what rate should he sell the mixer to gain 16%?

Sol **.**C.P of 200 kg of mixture = Rs. (80 * 13.50+120*16) = Rs.3000.

S.P =116% 0f Rs.3000 =Rs.[(116/200) *3000]=Rs.3480.

∴ Rate of S.P of the mixture =Rs.[3480/200] per kg =Rs.17.40 per kg.

Ex.13. Pure ghee cost Rs.100 per kg. After adulterating it with vegetable oil costing Rs.50 per kg, A shopkeeper sells the mixture at the rate of Rs.96 per kg, thereby making a profit of 20%.In What ratio does he mix the two?

Sol. Mean cost price =Rs. [(100/120)*96] =Rs.80 per kg.

By the rate of allegation :

C.P of 1kg ghee — C.P of 1kg oil

100 — 50

Mean price

80

30 — 20

$\therefore$ Required ratio =30:20 =3:2.

Ex. 14. A dishonest dealer professes to sell his goods at cost price but uses a weight of 960 gms for a kg weight . Find his gain percent.

Sol .Gain% $=\left[\dfrac{\text{Error}}{(\text{error value})-(\text{error})}*100\right]\% = [(40/960)*100]\ \% = 4\frac{1}{6}\ \%$

Ex 15. If the manufacturer gains 10%,the wholesale dealer 15% and the retailer 25% ,then find the cost of production of a ,the retail price of which is Rs.1265?

Sol:

Let the cost of production of the table be Rs x

The ,125% of 115% of 110% of x=1265

⇨ 125/100*115/100*110/100*x=1265=>253/160*x=>1265=>x=(1265 *160/253)=Rs.800

Ex16 . Monika purchased a pressure cooker at 9/10th of its selling price and sold it at 8% more than its S.P .find her gain percent.

Sol:

Let the s.p be Rs. X .then C.P = Rs.9x/10,Receipt=108% of rs.x=Rs 27x/25

Gain=Rs (27x/25*9x/10)=Rs(108x-90x/100)=Rs18x/100

Gain%=(18x/100*10/9x*100)%=20%

Ex .17 An article is sold at certain price. By selling it at 2/3 of its price one losses 10%,find the gain at original price ?

sol:

let the original s.p be Rs x. then now S.P=Rs2x/3,loss=10%

now C.P=Rs20x/27*27/20x*100)%=35%

Ex .18. A tradesman sold an article at a loss of 20%.if the selling price has been increased by Rs100,ther would have been a gain of 5%.what was the cost price of the article?

Sol:

Let C.P be Rs x. then (105% of x)-(80 % of x)=100 or 25% of x=100

⇨ x/4=100 or x=400

⇨ so,C.P =Rs 400

Ex 19. A man sells an article at a profit of 25%if he had bought it 20% less and sold it for Rs 10.50 less,he would have gained 30%find the cost price of the article.

Sol:

Let the C.P be Rs x

1^{st} S.P=125% of x =125x/100=5x/4;2^{nd} S.P=80% of x=80x/100=4x/5

2^{nd} S.P=130% of 4x/5=(130/100*4x/5)=26x/25

=>5x/4-26x/25=10.50⇔x=(10.50*100)/21=50

hence C.P=Rs.50

Ex 20.The price of the jewel,passing through three hands,rises on the whole by65%.if the first and the second sellers 20%and25% profit respectively find the percentage profit earned by the third seller.

Sol:

Let the orginal price of the jewel be Rs p and let the profit earned by the thrid seller be x%

Then,(100+x)% of 125% OF 120% OF P=165% OF P

⇨ ((100+X)/100*125/100*120/100*P)=(165/100*P)

⇨ (100+X)=(165*100*100)/(125*120)=110=>X=10%

Ex21 . A man 2 flats for Rs 675958 each.on one he gains 16% while on the other he losses 16%. How much does he gain/loss in the whole transaction?

Sol:

In this case there will be alwalys loss. The selling price is immaterial

Hence, loss % = (common loss and gain%)^2 /10=(16/10)%=(64/25)%=2.56%

Ex.22. A dealer sold three-fourth of his article at a gain of 20% and remaining at a cost price. Find the gain earned by him at the two transcation.

Sol:

Let the C.P of the whole be Rs x

C.P of 3/4th =Rs 3x/4,C.P of 1/4th =Rs x/4

⇨ total S.P=Rs [(120%of 3x/4)+x/4]=Rs(9x/10+x/4)=Rs 23x/20

⇨ gain=Rs(23x/20-x)=Rs 3x/20

⇨ gain%=3x/20*1/x*100)%=15%

Ex 23 ..A man bought a horse and a car riage for Rs 3000.he sold the horse at a gain of 20% and the carriage at a loss of 10%,thereby gaining 2% on the whole.find the cost of the horse.

Sol:

Let the C.p of the horse be Rs.x, then C.P of the carriage =Rs(3000-x)

20% of x-10% of(3000-x)=2% of 3000

⇨ x/5-(3000-x)/10=60=.2x-3000+x=600=.3x+3600=>x=1200

⇨ hence,C.P of the horse =Rs 1200

Ex 24 find the single discount equivalent to a series discount of 20% ,10% and 5%'

sol:

let the marked price be Rs 100

then ,net S.P=95% of 90% of 80% of Rs 100

=Rs(95/100*90/100*80/100*100)=Rs68.40

Ex .25 After getting 2 successive discounts, a shirt with a list price of Rs 150 is available at Rs 105. If the second discount is 12.55,find the first discount.

Sol:

Let the first discount be x%

Then,87.5% of (100-x)% of 150= 105

- 87.5/100*(100-x)/100*450=150=>105=>100-x=(105*100*100)/(150*87.5)=80
- x=(100-80)=20
- first discount = 20%

Ex .26 An uneducated retailar marks all its goods at 50% above the cost price and thinking that he will still make 25% profit,offers a discount of 25% on the marked price.what is the actual profit on the sales?

Sol:

Let C.P =Rs 100.then ,marked price =Rs100

S.P=75% of Rs 150=Rs112.50

Hence,gain%=12.50%

Ex27 .A retailer buys 40 pens at the market price of 36 pens from a wholesaler ,if he sells these pens giving a discount of 1% ,what is the profit % ?

sol:

let the market price of each pen be Rs 1

then,C.P of 40 pens = Rs 36 S.P of 40 pens =99% of Rs 40=Rs 39.60

profit %=((3.60*100)/36) %=10%

Ex 28 . At what % above C.P must an article be marked so as to gain 33% after allowing a customer a discount of 5%?

Sol

Let C.P be Rs 100.then S.P be Rs 133

Let the market price be Rs x

Then 90% of x=133=>95x/100=133=>x=(133*100/95)=140

Market price = 40% above C.P

Ex .29 . When a producer allows 36% commission on retail price of his product, he earns a profit of 8.8%. what would be his profit % if the commision is reduced by 24%?

Sol:

Let the retail price =Rs 100.then, commission=Rs 36

S.P=Rs(100-36)=Rs 64

But, profit=8.8%

C.P=Rs(100/108.8*64)=Rs 1000/17

New commission =Rs12. New S.P=Rs(100-12)Rs 88

Gain=Rs(88-1000/17)=Rs 496/17

Gain%=(496/17*17/1000*100)%=49.6%

12. RATIO AND PROPORTION

IMPORTANT FACTS AND FORMULAE

I. RATIO: The ratio of two quantities a and b in the same units, is the fraction a/b and we write it as a:b. In the ratio a:b, we call a as the **first term or antecedent** and b, the **second term or consequent.**

Ex. The ratio 5: 9 represents 5/9 with antecedent = 5, consequent = 9.
Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.
Ex. 4: 5 = 8: 10 = 12: 15 etc. Also, 4: 6 = 2: 3.

2. **PROPORTION:** ***The equality of two ratios is called proportion.***
If *a:* b = c: d, we write, *a:* b:: c : d and we say that *a,* b, c, d are in proportion . Here *a* and d are called extremes, while b and c are called mean terms.
Product of means = Product of extremes.
Thus, a: b:: c : d <=> (b x c) = (a x d).

3. (i) Fourth Proportional: If *a* : b = c: d, then d is called the fourth proportional to *a,* b, c.
(ii) Third Proportional: If *a*: b = b: c, then c is called the third proportional to a and b.
(iii) Mean Proportional: Mean proportional between *a* and *b* is *square root of ab*

4. (i) COMPARISON OF RATIOS:
We say that *(a*: *b)* > (c: *d)* <=> (a/b)>(c /d).
(ii) COMPOUNDED RATIO:
The compounded ratio of the ratios (a: *b),* (c: *d),* (e : f) is *(ace*: bdf)

5. (i) *Duplicate ratio* of *(a* : b) is $(a^2 : b^2)$.
(ii) Sub-duplicate ratio of (a : *b)* is $(\sqrt{a} : \sqrt{b})$.
(iii)Triplicate ratio of (a : *b)* is $(a^3 : b^3)$.
(iv) Sub-triplicate ratio of (a : *b)* is $(a^{1/3} : b^{1/3})$.
(v) If (a/b)=(c/d), then ((a+b)/(a-b))=((c+d)/(c-d)) **(Componendo and dividendo)**

6. VARIATION:
(i) We say that x is directly proportional to *y,* if x = *ky* for some constant k and we write, $x \propto y$.
(ii) We say that x is inversely proportional to *y,* if xy = *k* for some constant k and we write, x∞(1/y)

SOLVED PROBLEMS

Ex. 1. ***If* a : *b* = 5 : 9 *and* b : c = 4: 7, *find* a : b : c.**

Sol. a:b=5:9 and b:c=4:7= (4X9/4): (7x9/4) = 9:63/4
a:b:c = 5:9:63/4 =20:36:63.

Ex. 2. ***Find:***

(i) *the fourth proportional* to 4, 9, 12;
(ii) *the third proportional* to 16 *and* 36;
iii) the* mean *proportional between 0.08 and 0.18.

Sol.

i) Let the fourth proportional to 4, 9, 12 be x.

Then, 4 : 9 : : 12 : x ⇔4 x x=9x12 ⇔ X=(9 x 12)/14=27;
Fourth proportional to 4, 9, 12 is 27.

(ii) Let the third proportional to 16 and 36 be x.

Then, 16 : 36 : : 36 : x ⇔16 x *x* = 36 x 36 ⇔ x=(36 x 36)/16 =81
Third proportional to 16 and 36 is 81.

(iii) Mean proportional between 0.08 and 0.18
√0.08 x 0.18 =√8/100 x 18/100= √144/(100 x 100)=12/100=0.12

Ex. 3. ***If* x : *y* = 3 : 4, *find* (4x + *5y)* : (5x - *2y).***

Sol. X/Y=3/4 ⇔ (4x+5y)/(5x+2y)= (4(x/y)+5)/(5 (x/y)-2) =(4(3/4)+5)/(5(3/4)-2)

=(3+5)/(7/4)=32/7

Ex. 4. ***Divide Rs.* 672 *in the ratio* 5 : 3**.

Sol. Sum of ratio terms = (5 + 3) = 8.

First part = Rs. (672 x (5/8)) = Rs. 420; Second part = Rs. (672 x (3/8)) = Rs. 252.

Ex. 5. ***Divide Rs.*** **1162 among** ***A, B,*** **C** ***in the ratio*** **35 : 28 :** ***20.***

Sol. Sum of ratio terms = (35 + 28 + 20) = 83.

A's share = Rs. (1162 x (35/83))= Rs. 490; B's share = Rs. (1162 x(28/83))= Rs. 392;

C's share = Rs. (1162 x (20/83))= Rs. 280.

Ex. 6. ***A bag contains*** **50** ***p,*** **25** ***P and*** **10** ***p coins in the ratio*** **5: 9: 4,** ***amounting*** **to** ***Rs. 206. Find the*** **number of** ***coins*** **of** ***each type.***

Sol. Let the number of 50 p, 25 P and 10 p coins be 5x, 9x and 4x respectively.

(5x/2)+(9x/ 4)+(4x/10)=206 ⇔ 50x + *45x* + 8x = 4120⇔1O3x = 4120 ⇔x=40.

Number of 50 p coins = (5 x 40) = 200; Number of 25 p coins = (9 x 40) = 360;
Number of 10 p coins = (4 x 40) = 160.

Ex. 7. ***A*** **mixture** ***contains alcohol*** **and** ***water*** **in** ***the ratio*** **4 : 3.** ***If*** **5 litres of water** ***is*** **added** ***to the mixture, the ratio becomes*** **4: 5.** ***Find the quantity of alcohol*** **in the given mixture**

Sol. Let the quantity of alcohol and water be 4x litres and 3x litres respectively

4x/(3x+5)=4/5 ⇔20x=4(3x+5)⇔8x=20 ⇔x=2.5
Quantity of alcohol = (4 x 2.5) litres = 10 litres.

13. PARTNERSHIP

!IMPORTANT FACTS AND FORMULAE I

1. Partnership: When two or more than two persons run a business jointly, they are called partners and the deal is known as *partnership.*

2. Ratio of Division of Gains:

i) When investments of all the partners are for the same *time,* the gain or loss is distributed a among the partners in the ratio of their investments.
Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year:
(A's share of profit) : (B's share of profit) = x : y.

ii) ***When investments are for different time periods,*** then equivalent capitals are calculated for a unit of time by taking (capital x number of units of time). Now, gain or loss is divided in the ratio of these capitals.

Suppose A invests Rs. x for p months and B invests Rs. y for q months, then
(A's share of profit) : (B's share of profit) = xp : *yq.*

3. Working and Sleeping Partners: A partner who manages the business is known . as a *working partner* and the one who simply invests the money is a *sleeping partner.*

SOLVED EXAMPLES

Ex. 1. A, B and C started a business by investing Rs. 1,20,000, Rs. 1,35,000 and ,Rs.1,50,000 respectively. Find the share of each, out of an annual profit of Rs. 56,700.

Sol. Ratio of shares of A, Band C = Ratio of their investments
= 120000 : 135000 : 150000 = 8 : 9 : 10.

A's share = Rs. (56700 x (8/27))= Rs. 16800.

B's share = Rs. (56700 x (9/27)) = Rs. 18900.

C's share = Rs. (56700 x (10/27))=Rs. 21000.

Ex. 2. Alfred started a business investing Rs. 45,000. After 3 months, Peter joined him with a capital of Rs. 60,000. After another 6 months, Ronald joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 16,500. Find the lire of each.

Sol. Clearly, Alfred invested his capital for 12 months, Peter for 9 months and Ronald for 3 months.

So, ratio of their capitals = (45000 x 12) : (60000 x 9) : (90000 x 3)
= 540000 : 540000 : 270000 = 2 : 2 : 1.

Alfred's share = Rs. (16500 x (2/5)) = Rs. 6600

Peter's share = Rs. (16500 x (2/5)) = Rs. 6600

Ronald's share = Rs. (16500 x (1/5)) = Rs. 3300.

Ex. 3. A, Band C start a business each investing Rs. 20,000. After 5 months A withdrew Rs.6000 B withdrew Rs. 4000 and C invests Rs. 6000 more. At the end of the year, a total profit of Rs. 69,900 was recorded. Find the share of each.

Sol. Ratio of the capitals of A, Band C
= 20000 x 5 + 15000 x 7 : 20000 x 5 + 16000 x 7 : 20000 x 5 + 26000 x 7
= 205000:212000 : 282000 = 205 : 212 : 282.

A's share = Rs. 69900 x (205/699) = Rs. 20500 I

B's share = Rs. 69900 x (212/699) = Rs. 21200;

C's share = Rs. 69900 x (282/699) = Rs. 28200.

Ex. 4. A, Band C enter into partnership. A invests 3 times as much as B and B invests two-third of what C invests. At the end of the year, the profit earned is Rs. 6600. What is the share of B ?

Sol. Let C's capital = Rs. x. Then, B's capital = Rs. (2/3)x

A's capital = Rs. (3 x (2/3).*x*) = Rs. *2x.*

Ratio of their capitals = *2x* : (2/3)x :x = 6 : 2 : 3.

Hence, B's share = Rs. (6600 x (2/11))= Rs. 1200.

Ex. 5. Four milkmen rented a pasture. A grazed 24 cows for 3 months; B 10 for 5 months; C 35 cows for 4 months and D 21 cows for 3 months. If A's share of rent is Rs. 720, find the total rent of the field.

Sol. Ratio of shares of A, B, C, D = (24 x 3) : (10 x 5) : (35 x 4) : (21 x 3) = 72 : 50 : 140 : 63.

Let total rent be Rs. x. Then, *A's* share = Rs. (72x)/325

$(72x)/325=720 \Leftrightarrow x=(720 \times 325)/72 = 3250$

Hence, total rent of the field is Rs. 3250.

Ex.6. A invested Rs. 76,000 in a business. After few months, B joined him Rs. 57,000. At the end of the year, the total profit was divided between them in ratio 2 : 1. After bow many months did B join?

Sol. Suppose B joined after x months. Then, B's money was invested for (12 - x)

$(76000 \times 12)/(57000 \times (12-x) =2/1 \Leftrightarrow 912000=114000(12-x)$

$114 (12 - x) = 912 \Leftrightarrow 12-x=8 \Leftrightarrow x=4$

Hence, B joined after 4 months.

Ex.7. A, Band C enter into a partnership by investing in the ratio of 3 : 2: 4. After 1 year, B invests another Rs. 2,70,000 and C, at the end of 2 years, also invests Rs.2,70,000. At the end of three years, profits are shared in the ratio of 3 : 4 : 5. Find initial investment of each.

Sol. Let the initial investments of A, Band C be Rs. *3x,* Rs. 2x and Rs. 4x respectively.

Then,

$(3x \times 36) : [(2x \times 12) + (2x + 270000) \times 24] : [(4x \times 24) + (4x +270000) \times 12]=3:4:5$

$1O8x : (72x + 6480000) : (144x + 3240000) = 3 : 4 : 5$

$108x /(72x+6480000)=3/4 \Leftrightarrow 432x = 216x + 19440000$

$\Leftrightarrow 216x = 19440000$

$x=90000$

Hence, A's initial investment = 3x = Rs. 2,70,000;

B's initial investment = 2x = Rs. 1,80,000;

C's initial investment = 4x = Rs. 3,60,000.

14. CHAIN RULE

_IMPORTANT FACTS AND FORMULAE

1. Direct Proportion: Two quantities are said to be directly proportional, if on the increase (or decrease) of the one, the other increases (or decreases) to the same

Ex. 1. Cost is directly proportional to the number of articles.
(More Articles, More Cost)

Ex. 2. Work done is directly proportional to the number of men working on it
(More Men, More Work)

2. Indirect Proportion: Two quantities are said to be indirectly proportional,if on the increase of the one, the other decreases to the same extent and vice-versa.

Ex. 1. The time taken by a car in covering a certain distance is inversely proportional to the speed of the car.
(More speed, Less is the time taken to cover a distance)

Ex. 2. Time taken to finish a work is inversely proportional to the num of persons working at it.
(More persons, Less is the time taken to finish a job)

Remark: In solving questions by chain rule, we compare every item with the term to be found out.

SOL VED EXAMPLES

Ex. 1. If 15 toys cost Rs, 234, what do 35 toys cost?

Sol. Let the required cost be Rs. x. Then,
More toys, More cost (Direct Proportion)

. 15 : 35 : : 234 : x ⇔ (15 x x) = (35 x 234) ⇔ x=(35 X 234)/15 =546

Hence, the cost of 35 toys is Rs. 546.

Ex. 2. If 36 men can do a piece of work in 25 hours, in how many hours will 15 men do it ?

Sol. Let the required number of hours be x. Then,
Less men, More hours (Indirect Proportion)

15 : 36 : : 25 : x ⇔(15 x x) = (36 x 25) ⇔(36 x 25)/15 = 60

Hence, 15 men can do it in 60 hours.

Ex. 3. If the wages of 6 men for 15 days be Rs.2100, then find the wages of for 12 days.

Sol. Let the required wages be Rs. x.

More men, More wages (Direct Proportion)
Less days, Less wages (Direct Proportion)

Men 6: 9 : :2100:x
Days 15:12

Therefore (6 x 15 x x)=(9 x 12 x 2100) ⇔ x=(9 x 12 x 2100)/(6 x 15)=2520

Hence the required wages are Rs. 2520.

Ex. 4. If 20 men can build a wall 66 metres long in 6 days, what length of a similar can be built by 86 men in 8 days?

Sol. Let the required length be x metres

More men, More length built (Direct Proportion)

Less days, Less length built (Direct Proportion)

Men 20: 35
Days 6: 3 : : 56 : x

Therefore (20 x 6 x x)=(35 x 3 x 56)⇔x=(35 x 3 x 56)/120=49

Hence, the required length is 49 m.

Ex. 5. If 15 men, working 9 hours a day, can reap a field in 16 days, in how many days will 18 men reap the field, working 8 hours a day?

Sol. Let the required number of days be x.
More men, Less days (indirect proportion)
Less hours per day, More days (indirect proportion)

Men 18 : 15
Hours per day 8: 9 } : :16 : x
(18 x 8 x x)=(15 x 9 x 16)⇔ x=(44 x 15)144 = 15

Hence, required number of days = 15.

Ex. 6. If 9 engines consume 24 metric tonnes of coal, when each is working 8 hours day, bow much coal will be required for 8 engines, each running 13hours a day, it being given that 3 engines of former type consume as much as 4 engines of latter type?

Sol. Let 3 engines of former type consume 1 unit in 1 hour.
Then, 4 engines of latter type consume 1 unit in 1 hour.

Therefore 1 engine of former type consumes(1/3) unit in 1 hour.

1 engine of latter type consumes(1/4) unit in 1 hour.

Let the required consumption of coal be x units.
Less engines, Less coal consumed (direct proportion)
More working hours, More coal consumed (direct proportion)
Less rate of consumption, Less coal consumed(direct prportion)

Number of engines 9: 8
Working hours 8 : 13 } :: 24 : x
Rate of consumption (1/3):(1/4)

[9 x 8 x (1/3) x x) = (8 x 13 x (1/4) x 24) ⇔ 24x = 624 ⇔ x = 26.

Hence, the required consumption of coal = 26 metric tonnes.

Ex. 7. A contract is to be completsd in 46 days sad 117 men were said to work 8 hours a day. After 33 days, (4/7) of the work is completed. How many additional men may be employed so that the work may be completed in time, each man now working 9 hours a day?

Sol. Remaining work = (1-(4/7) =(3/7)

Remaining period = (46 - 33) days = 13days

Let the total men working at it be x.

Less work, Less men	(Direct Proportion)
Less days, More men	(Indirect Proportion)
More Hours per Day, Less men	(Indirect Proportion)

Work (4/7): (3/7)
Days 13:33 } : : 117: x
Hrs/day 9 : 8
Therefore (4/7) x 13 x 9 x x =(3/7) x 33 x 8 x 117 or x=(3 x 33 x 8 x 117)/(4 x 13 x 9)=198

Additional men to be employed = (198 - 117) = 81.

Ex. 8. A garrison of 3300 men had provisions for 32 days, when given at the rate of 860 gns per head. At the end of 7 days, a reinforcement arrives and it was for that the provisions wi1l last 17 days more, when given at the rate of 826 gms per head, What is the strength of the reinforcement?

Sol. The problem becomes:
3300 men taking 850 gms per head have provisions for (32 - 7) or 25 days,
How many men taking 825 gms each have provisions for 17 days?

Less ration per head, more men	(Indirect Proportion)
Less days, More men	(Indirect Proportion)

Ration 825 : 850

Days 17: 25 } : : 3300 : x

$(825 \times 17 \times x) = 850 \times 25 \times 3300$ or $x = (850 \times 25 \times 3300)/(825 \times 17) = 5000$

Strength of reinforcement = (5500 - 3300) = 1700.

15. TIME AND WORK

IIMPORTANT FACTS AND FORMULAE

1. If A can do a piece of work in n days, then A's 1 day's work = (1/n).

2. If A’s 1 day's work = (1/n),then A can finish the work in n days.

3. A is thrice as good a workman as B, then:
   Ratio of work done by A and B = 3 : 1.
   Ratio of times taken by A and B to finish a work = 1 : 3.

SOLVED EXAMPLES

Ex. 1. Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same Job.How long should it take both A and B, working together but independently, to do the same job? (IGNOU, 2003)

Sol. A’s 1 hour's work = 1/8

B's 1 hour's work = 1/10

(A + B)'s 1 hour's work = (1/8) +(1/10)=9/40

Both A and B will finish the work in 40/9 days.

Ex. 2. A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work? (Bank P.O. 2003)

Sol. (A + B)'s 1 day's work = (1/4). A's 1 day's work = (1/12).

B's 1 day's work =((1/4)-(1/12))=(1/6)

Hence, B alone can complete the work in 6 days.

Ex. 3. A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days

of 7 bours each. How long will they take to do it, working together 8 hours a day?

Sol. A can complete the work in (7 x 9) = 63 hours.
B can complete the work in (6 x 7) = 42 hours.

A's 1 hour's work = (1/63) and B's 1 hour's work =(1/42)

(A + B)'s 1 hour's work =(1/63)+(1/42)=(5/126)

Both will finish the work in (126/5) hrs.
Number of days. of (42/5) hrs each =(126 x 5)/(5 x 42)=3 days

Ex. 4. A and B can do a piece of work in 18 days; Band C can do it in 24 days A and C can do it in 36 days. In how many days will A, Band C finish it together and separately?

Sol. (A + B)'s 1 day's work = (1/18)   (B + C)'s 1 day's work = (1/24)
and (A + C)'s 1 day's work = (1/36)

Adding, we get: 2 (A + B + C)'s 1 day's work =(1/18 + 1/24 + 1/36)

=9/72 =1/8

(A +B + C)'s 1 day's work =1/16

Thus, A, Band C together can finish the work in 16 days.
Now, A's 1 day's work = [(A + B + C)'s 1 day's work] - [(B + C)'s 1 day work:
=(1/16 – 1/24)= 1/48

A alone can finish the work in 48 days.

Similarly, B's 1 day's work =(1/16 – 1/36)=5/144

B alone can finish the work in 144/5=28 4/5 days

And C's 1 day work =(1/16-1/18)=1/144

Hence C alone can finish the work in 144 days.

Ex. 6. A is twice as good a workman as B and together they finish a piece in 18 days. In how many days will A alone finish the work?
Sol. (A's 1 day's work):)(B's 1 days work) = 2 : 1.

(A + B)'s 1 day's work = 1/18

Divide 1/18 in the ratio 2 : 1.

:. A’s 1 day's work =(1/18*2/3)=1/27

Hence, A alone can finish the work in 27 days.

Ex. 6. A can do a certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?

Sol. Ratio of times taken by A and B = 160 : 100 = 8 : 5.

Suppose B alone takes x days to do the job.

Then, 8 : 5 :: 12 : x = 8x = 5 x 12 =x = 7 1/2 days.

Ex. 7. A can do a piece of work in 80 days. He works at it for 10 days B alone finishes the remaining work in 42 days. In how much time will A and B working together, finish the work?

Sol. Work done by A in 10 days =(1/80*10)=1/8

Remaining work = (1- 1/8) =7/ 8

Now,7/ 8 work is done by B in 42 days.

Whole work will be done by B in (42 x 8/7) = 48 days.

A’s 1 day's work = 1/80 and B's 1 day's work = 1/48

(A+B)'s 1 day's work = (1/80+1/48)=8/240=1/30

Hence, both will finish the work in 30 days.

Ex. 8. A and B undertake to do a piece of work for Rs. 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. !find the share of each.

Sol :C's 1 day's work = 1/3-(1/6+1/8)=24

A : B : C = Ratio of their 1 day's work = 1/6:1/8:1/24= 4 : 3 : 1.

A’s share = Rs. (600 *4/8) = Rs.300, B's share = Rs. (600 *3/8) = Rs. 225.

C's share = Rs. [600 - (300 + 225») = Rs. 75.

Ex. 9. A and B working separately can do a piece of work in 9 and 12 days respectively, If they work for a day alternately, A beginning, in how many days, the work will be completed?

(A + B)'s 2 days' work =(1/9+1/12)=7/36

Work done in 5 pairs of days =(5*7/36)=35/36

Remaining work =(1-35/36)=1/36

On 11th day, it is A’s turn. 1/9 work is done by him in 1 day.

1/36 work is done by him in(9*1/36)=1/4 day

Total time taken = (10 + 1/4) days = 10 1/4days.

Ex 10 .45 men can complete a work in 16 days. Six days after they started working, 30 more men joined them. How many days will they now take to complete the remaining work?

(45 x 16) men can complete the work in 1 day.

1 man's 1 day's work = 1/720

45 men's 6 days' work =(1/16*6)=3/8

Remaining work =(1-3/8)=5/8

75 men's 1 day's work = 75/720=5/48

Now, $\frac{5}{48}$ work is done by them in 1 day.

$\frac{5}{8}$ work is done by them in $\left(\frac{48}{5} \times \frac{5}{8}\right)$=6 days.

Ex:11. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days.In how many days can 2 men and 1 boy do the work?

Soln: Let 1 man's 1 day's work = x and 1 boy's 1 day's work = y.

Then, $2x+3y = \frac{1}{10}$ and $3x+2y = \frac{1}{8}$

Solving,we get: $x = \frac{7}{200}$ and $y = \frac{1}{100}$

(2 men + 1 boy)'s 1 day's work $= \left(2 \times \frac{7}{200} + 1 \times \frac{1}{100}\right) = \frac{16}{200} = \frac{2}{25}$

So, 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12\frac{1}{2}$ days

16. PIPES AND CISTERNS

IMPORTANT FACTS AND FORMULAE

1. **Inlet:** A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.
Outlet: A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

2. (i) If a pipe can fill a tank in x hours, then : part filled in 1 hour = 1/x

(ii) If a pipe can empty a full tank in *y* hours, then : part emptied in 1 hour = 1/y

(iii) If a pipe can .fill a tank in x hours and another pipe can empty the full tank in y hours (where *y> x),* then on opening both the pipes, the net part filled in 1 hour = (1/x)-(1/y)

(iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where x > *y),* then on opening both the pipes, the net part emptied in 1 hour = (1/y)-(1/x)

SOLVED EXAMPLES

Ex. 1:Two pipes A and B can fill a tank in 36 bours and 46 bours respectively. If both the pipes are opened simultaneously, bow mucb time will be taken to fill the tank?

Sol: Part filled by A in 1 hour = (1/36);

Part filled by B in 1 hour = (1/45);

Part filled by (A + B) In 1 hour =(1/36)+(1/45)=(9/180)=(1/20)

Hence, both the pipes together will fill the tank in 20 hours.

Ex. 2: Two pipes can fill a tank in 10hours and 12 hours respectively while a third, pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled?

Sol: Net part filled In 1 hour =(1/10)+(1/12)-(1/20)=(8/60)=(2/15).
The tank will be full in 15/2 hrs = 7 hrs 30 min.

Ex. 3: If two pipes function simultaneously, tbe reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than tbe otber. How many hours does it take the second pipe to fill the reservoir?

Sol:let the reservoir be filled by first pipe in x hours.

Then ,second pipe fill it in (x+10)hrs.

Therefore (1/x)+(1/x+10)=(1/12) ⇔(x+10+x)/(x(x+10))=(1/12).

⇔ x^2 –14x-120=0 ⇔ (x-20)(x+6)=0

⇔x=20 [neglecting the negative value of x]

so, the second pipe will take (20+10)hrs. (i.e) 30 hours to fill the reservoir

Ex. 4: A cistern has two taps which fill it in 12 minutes and 15minutes respectively. There is also a waste pipe in the cistern. When all the 3 are opened , the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Sol: Workdone by the waste pipe in 1min

=(1/20)-(1/12)+(1/15) = -1/10 [negative sign means emptying]

therefore the waste pipe will empty the full cistern in 10min

Ex. 5: An electric pump can fill a tank in 3 hours. Because of a leak in ,the tank it took 3(1/2) hours to fill the tank. If the tank is full, how much time will the leak take to empty it ?

Sol: work done by the leak in 1 hour=(1/3)-(1/(7/2))=(1/3)-(2/7)=(1/21).

The leak will empty .the tank in 21 hours.

Ex. 6. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it tooki 32 minutes more to fill the cistern.When the cistern is full, in what time will the leak empty it?

Sol: Work done by the two pipes in 1 hour =(1/14)+(1/16)=(15/112).

Time taken by these pipes to fill the tank = (112/15) hrs = 7 hrs 28 min.

Due to leakage, time taken = 7 hrs 28 min + 32 min = 8 hrs

Work done by (two pipes + leak) in 1 hour = (1/8).

Work done by the leak m 1 hour =(15/112)-(1/8)=(1/112).

Leak will empty the full cistern in 112 hours.

Ex. 7: Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened. after 7 min,C is also opened. In how much time, the tank is full?

Sol: Part filled in 7 min. = 7*((1/36)+(1/45))=(7/20).

Remaining part=(1-(7/20))=(13/20).

Net part filled in 1min. when A,B and C are opened=(1/36)+(1/45)-(1/30)=(1/60).

Now,(1/60) part is filled in one minute.

(13/20) part is filled in (60*(13/20))=39 minutes.

Ex.8: Two pipes A,B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 min.?

Sol: let B be closed after x min. then ,

Part filled by (A+B) in x min. +part filled by A in (18-x)min.=1

Therefore x*((1/24)+(1/32))+(18-x)*(1/24)=1 ⇔ (7x/96) + ((18-x)/24)=1.

⇔ 7x +4*(18-x)=96.

Hence, be must be closed after 8 min.

17. TIME AND DISTANCE

IMPORTANT FACTS AND FORMULAE

1. Speed = $\left(\frac{\text{Distance}}{\text{Time}}\right)$, Time= $\left(\frac{\text{Distance}}{\text{Speed}}\right)$, Distance = (Speed * Time)

2. x km / hr = $x * \frac{5}{18}$

3. x m/sec = (x * 18/5) km /hr

4. If the ratio of the speeds of A and B is a:b , then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or b:a.

5. Suppose a man covers a certain distance at x km/ hr and an equal distance at y km / hr . Then , the average speed during the whole journey is $\frac{2xy}{x+y}$ km/ hr.

SOLVED EXAMPLES

Ex. 1. How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr?

Sol. Aditya's speed = 20 km/hr = $\{20 * \frac{5}{18}\}$ m/sec = $\frac{50}{9}$ m/sec

$\therefore$ Time taken to cover 400 m= $\{400 * \frac{9}{50}\}$ sec =72 sec = $1\frac{12}{60}$ min $1\frac{1}{5}$ min.

Ex. 2. A cyclist covers a distnce of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?

Sol. Speed = $\{\frac{750}{150}\}$ m/sec =5 m/sec = $\{5 * \frac{18}{5}\}$ km/hr =18km/hr

Ex. 3. A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Compare their speeds.

Sol. Let the distance covered in 1 leap of the dog be x and that covered in 1 leap of the hare by y.

Then , 3x = 4y => x = $\frac{4}{3}$ y => 4x = $\frac{16}{3}$ y.

$\therefore$ Ratio of speeds of dog and hare = Ratio of distances covered by them in the same time

= 4x : 5y = $\frac{16}{3}$ y : 5y = $\frac{16}{3}$: 5 = 16:15

Ex. 4. While covering a distance of 24 km, a man noticed that after walking for 1 hour and 40 minutes, the distance covered by him was $\frac{5}{7}$ of the remaining distance. What was his speed in metres per second?

Sol. Let the speed be x km/hr.

Then, distance covered in 1 hr. 40 min. i.e., $1\frac{2}{3}$ hrs $= \frac{5x}{3}$ km

Remaining distance $= \left\{ 24 - \frac{5x}{3} \right\}$ km.

$$\therefore \frac{5x}{3} = \frac{5}{7}\left\{ 24 - \frac{5x}{3} \right\} \Leftrightarrow \frac{5x}{3} = \frac{5}{7}\left\{ \frac{72-5x}{3} \right\} \Leftrightarrow 7x = 72 - 5x$$

$$\Leftrightarrow 12x = 72 \Leftrightarrow x=6$$

Hence speed = 6 km/hr $=\left\{ 6 * \frac{5}{18} \right\}$ m/sec $= \frac{5}{3}$ m/sec $= 1\frac{2}{3}$

Ex. 5. Peter can cover a certain distance in 1 hr. 24 min. by covering two-third of the distance at 4 kmph and the rest at 5 kmph. Find the total distance.

Sol. Let the total distance be x km . Then,

$$\frac{\frac{2}{3}x}{4} + \frac{\frac{1}{3}x}{5} = \frac{7}{5} \Leftrightarrow \frac{x}{6} + \frac{x}{15} = \frac{7}{5} \Leftrightarrow 7x = 42 \Leftrightarrow x = 6$$

Ex. 6. A man traveled from the village to the post-office at the rate of 25 kmph and walked back at the rate of 4 kmph. If the whole journey took 5 hours 48 minutes, find the distance of the post-office from the village.

Sol. Average speed $= \left\{ \frac{2xy}{x+y} \right\}$ km/hr $=\left\{ \frac{2*25*4}{25+4} \right\}$ km/hr $= \frac{200}{29}$ km/hr

Distance traveled in 5 hours 48 minutes i.e., $5\frac{4}{5}$ hrs. $= \left\{ \frac{200}{29} * \frac{29}{5} \right\}$ km = 40 km

Distance of the post-office from the village $=\left\{ \frac{40}{2} \right\}$ = 20 km

Ex. 7. An aeroplane files along the four sides of a square at the speeds of 200,400,600 and 800km/hr. Find the average speed of the plane around the field.

Sol. :

Let each side of the square be x km and let the average speed of the plane around the field by y km per hour then ,

x/200+x/400+x/600+x/800=4x/y⇔25x/2500⇔4x/y⇔y=(2400*4/25)=384

hence average speed =384 km/hr

Ex. 8. Walking at $\frac{5}{7}$ of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.

Sol. : New speed =5/6 of the usual speed

New time taken=6/5 of the usual time

So,(6/5 of the usual time)-(usual time)=10 minutes.
=>1/5 of the usual time=10 minutes.

- ⇨ usual time=10 minutes

Ex. 9.If a man walks at the rate of 5 kmph, he misses a train by 7 minutes. However, if he walks at the rate of 6 kmph, he reaches the station 5 minutes before the arrival of the train. Find the distance covered by him to reach the station.
Sol. Let the required distance be x km
Difference in the time taken at two speeds=1 min =1/2 hr
Hence x/5-x/6=1/5<=>6x-5x=6
⇔x=6
Hence, the required distance is 6 km

Ex. 10. A and B are two stations 390 km apart. A train starts from A at 10 a.m. and travels towards B at 65 kmph. Another train starts from B at 11 a.m. and travels towards A at 35 kmph. At what time do they meet?
Sol. Suppose they meet x hours after 10 a.m. Then,
(Distance moved by first in x hrs) + [Distance moved by second in (x-1) hrs]=390.

$65x + 35(x-1) = 390 \Rightarrow 100x = 425 \Rightarrow x = 17/4$

So, they meet 4 hrs.15 min. after 10 a.m i.e., at 2.15 p.m.

Ex. 11. A goods train leaves a station at a certain time and at a fixed speed. After ^hours, an express train leaves the same station and moves in the same direction at a uniform speed of 90 kmph. This train catches up the goods train in 4 hours. Find the speed of the goods train.
Sol. Let the speed of the goods train be x kmph.
Distance covered by goods train in 10 hours= Distance covered by express train in 4 hours
$10x = 4 \times 90$ or $x = 36$.
So, speed of goods train = 36kmph.

Ex. 12. A thief is spotted by a policeman from a distance of 100 metres. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 8km/hr and that of the policeman 10 km/hr, how far the thief will have run before he is overtaken?
Sol. Relative speed of the policeman = (10-8) km/hr =2 km/hr.
Time taken by police man to cover 100m $\left(\frac{100}{1000} \times \frac{1}{2}\right)$ hr $= \frac{1}{20}$ hr.
In $\frac{1}{20}$ hrs, the thief covers a distance of $8 \times \frac{1}{20}$ km $= \frac{2}{5}$ km = 400 m

Ex.13. I walk a certain distance and ride back taking a total time of 37 minutes. I could walk both ways in 55 minutes. How long would it take me to ride both ways?

Sol. Let the distance be x km. Then,

(Time taken to walk x km) + (time taken to ride x km) =37 min.

(Time taken to walk 2x km) + (time taken to ride 2x km)= 74 min.

But, the time taken to walk 2x km = 55 min.

Time taken to ride 2x km = (74-55)min =19 min.

18. PROBLEMS ON TRAINS

IMPORTANT FACTS AND FORMULAE

1. *a km/hr*= (*a** 5/18) m/s.

2. a m / *s* = (a*18/5) km/hr.

3 Time taken by a train of length *1* metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover *1* metres.

4. Time taken by a train of length *1* metres to pass a stationary object of length *b* metres is the time taken by the train to cover *(1 + b)* metres.

5. Suppose two trains or two bodies are moving in the same direction at u m / s and *v m/s,* where u > *v,* then their relatives speed = (u - *v)* m / s.

6. Suppose two trains or two bodies are moving in opposite directions at u m / s and *v m/s,* then their relative speed is = (u + *v)* m/s.

7. If two trains of length *a* metres and *b* metres are moving in opposite directions at u m / s and *v* m/s, then time taken by the trains to cross each other = *(a + b)/(u+v)* sec.

8.If two trains of length a metres and *b* metres are moving in the same direction at u m / s and *v* m / s, then the time taken by the faster train to cross the slower train = (a+b)/(u-v) sec.

9. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take *a* and *b* sec in reaching B and A respectively, then

(A's speet) : *(B's speed)* = $(b^{1/2} : a^{1/2})$.

SOLVED EXAMPLES

Ex.I. A train 100 m long is running at the speed of 30 km / hr. Find the time taken by it to pass a man standing near the railway line. (S.S.C. 2001)

Sol. Speed of the train = (30 x 5/18) m / sec = (25/3) m/ sec.

Distance moved in passing the standing man = 100 m.

Required time taken = *100/(25/3)* = (100 *(3/25)) sec = 12 sec

Ex. 2. A train is moving at a speed of 132 km/br. If the length of the train is 110 metres, how long will it take to cross a railway platform 165 metres long? (Section Officers', 2003)

Sol. Speed of train = 132 *(5/18) m/sec = 110/3 m/sec.

Distance covered in passing the platform = (110 + 165) m = 275 m.

Time taken =275 *(3/110) sec =15/2 sec = 7 ½ sec

Ex. 3. A man is standing on a railway bridge which is 180 m long. He finds that a train crosses the bridge in 20 seconds but himself in 8 seconds. Find the length of the train and its speed?

Sol. Let the length of the train be x metres,

Then, the train covers x metres in 8 seconds and (x + 180) metres in 20 sec

$x/8=(x+180)/20 \Leftrightarrow 20x = 8(x + 180) \iff x = 120.$

Length of the train = 120 m.

Speed of the train = (120/8) m / sec = m / sec = (15 *18/5) kmph = 54 km

Ex. 4. A train 150 m long is running with a speed of 68 kmph. In what time will it pass a man who is running at 8 kmph in the same direction in which the train is going?

Sol: Speed of the train relative to man = (68 - 8) kmph

= (60* 5/18) m/sec = (50/3)m/sec

Time taken by the train to cross the man I

= Time taken by It to cover 150 m at 50/3 m / sec = 150 *3/ 50 sec = 9sec

Ex. 5. A train 220 m long is running with a speed of 59 kmph.. In what will

it pass a man who is running at 7 kmph in the direction opposite to that in which the train is going?

sol. Speed of the train relative to man = (59 + 7) kmph

= 66 *5/18 *m/sec* = 55/3 m/sec.

Time taken by the train to cross the man = Time taken by it to cover 220 m at (55/3) m / sec = (220 *3/55) sec = 12 sec

Ex. 6. Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmpb. In what time will they be clear of each other from the moment they meet?

Sol. Relative speed of the trains = (42 + 48) kmph = 90 kmph

=(90*5/18) m / sec = 25 m /sec.

Time taken by the trains to'pass each other

= Time taken to *cover* (137 + 163) m at 25 m */sec* =(300/25) sec = 12 sec

Ex. 7. Two trains 100 metres and 120 metres long are running in the same direction with speeds of 72 km/hr,In howmuch time will the first train cross the second?

Sol: Relative speed of the trains = (72 - 54) km/hr = 18 *km/hr*

= (18 * 5/18) *m/sec* = 5 *m/sec.*

Time taken by the trains to cross each other

= Time taken to cover (100 + 120) m at 5 m /sec = (220/5) sec = 44 sec.

Ex. 8. A train 100 metres long takes 6 seconds to cross a man walking at 5 kmph in the direction opposite to that of the train. Find the speed of the

train.?

Sol:Let the speed of the train be x kmph.

Speed of the train relative to man = (x + 5) kmph = $(x + 5) * 5/18$ m/sec.

Therefore $100/((x+5)*5/18)=6 \Leftrightarrow 30(x + 5) = 1800 \Leftrightarrow x = 55$

Speed of the train is 55 kmph.

Ex9. A train running at 54 kmph takes 20 seconds to pass a platform. Next it takes.12 sec to pass a man walking at 6 kmph in the same direction in which the train is going . Find the length of the train and the length of the platform.

Sol: Let the length of train be x metres and length of platform be y metres.

Speed of the train relative to man = (54 - 6) kmph = 48 kmph

= 48*(5/18) m/sec = 40/3 m/sec.

In passing a man, the train covers its own length with relative speed.

Length of train = (Relative speed * Time) = (40/3)*12 m = 160 m.

Also, speed of the train = 54 *(5/18)m / sec = 15 m / sec.

$(x+y)/15 = 20 \Leftrightarrow x + y = 300 \Leftrightarrow Y = (300 - 160)$ m = 140 m.

Ex10. A man sitting in a train which is traveling at 50 kmph observes that a goods train, traveling in opposite direction, takes 9 seconds to pass him. If the goods train is 280 m long, find its speed.?

Sol: Relative speed = 280/9 m / sec = ((280/9)*(18/5)) kmph = 112 kmph.

Speed of goods train = (112 - 50) kmph = 62 kmph.

19.BOATS AND STREAMS

IMPORTANT FACTS AND FORMULAE

1.In water ,the direction along the stream is called downstream and ,the direction against the stream is called upstream.

2.If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr,then:

speed downstream=(u+v)km/hr.
speed upstream=(u-v)km/hr.

3.If the speed downstream is a km/hr and the speed upstream is b km/hr,then :
speed in still water=1/2(a+b)km/hr
rate of stream=1/2(a-b)km/hr

SOLVED EXAMPLES

EX.1.A man can row upstream at 7 kmph and downstream at 10kmph.find man's rate in still water and the rate of current.

Sol. Rate in still water=1/2(10+7)km/hr=8.5 km/hr.
Rate of current=1/2(10-7)km/hr=1.5 km/hr.

EX.2. A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 2hours30minutes to cover a distance of 5km upstream. find the speed of the river current in km/hr.

Sol. rate downstream=(15/3 ¾)km/hr=(15*4/15)km/hr=4km/hr.
Rate upstream=(5/2 ½)km/hr=(5*2/5)km/hr=2km/hr.
Speed of current=1/2(4-2)km/hr=1km/hr

EX.3. a man can row 18 kmph in still water.it takes him thrice as long to row up as to row down the river.find the rate of stream.

Sol. Let man's rate upstream be x kmph.then ,his rate downstream=3xkmph.
So,2x=18 or x=9.
Rate upstream=9 km/hr,rate downstream=27 km/hr.
Hence,rate of stream=1/2(27-9)km/hr=9 km/hr.

EX.4. there is a road beside a river.two friends started from a place A,moved to a temple situated at another place B and then returned to A again.one of them moves

on a cycle at a speed of 12 km/hr,while the other sails on a boat at a speed of 10 km/hr.if the river flows at the speed of 4 km/hr,which of the two friends will return to placeA first?

Sol. Clearly the cyclist moves both ways at a speed of 12 km/hr.

The boat sailor moves downstream @ (10+4)i.e.,14 km/hr and upstream @ (10-4)i.e., 6km/hr.

So,average speed of the boat sailor=(2*14*6/14+6)km/hr

=42/5 km/hr=8.4 km/hr.

since the average speed of the cyclist is greater ,he will return ta A first.

EX.5. A man can row 7 ½ kmph in still water.if in a river running at 1.5 km/hr an hour,it takes him 50 minutes to row to a place and back,how far off is the place?

Sol. Speed downstream =(7.5+1.5)km/hr=9 km/hr;

Speed upstream=(7.5-1.5)kmph=6kmph.

Let the required distance be x km.then,

x/9+x/6=50/60.

2x+3x=(5/6*18)

5x=15

x=3.

Hence,the required distance is 3km.

EX.6. In a stream running at 2kmph,a motar boat goes 6km upstream and back again to the starting point in 33 minutes.find the speed of the motarboat in still water.

Sol.let the speed of the motarboat in still water be x kmph.then,

6/x+2 +6/x-2=33/60

$11x^2$-240x-44=0

$11x^2$-242x+2x-44=0

(x-22)(11x+2)=0

x=22.

EX.7.A man can row 40km upstream and 55km downstream in 13 hours also, he can row 30km upstream and 44km downstream in 10 hours.find the speed of the man in still water and the speed of the current.

Sol.let rate upstream=x km/hr and rate downstream=y km/hr.

Then,40/x +55/y =13…(i) and 30/x +44/y =10

Multiplying (ii) by 4 and (i) by 3 and subtracting ,we get:11/y=1 or y=11.

Substituting y=11 in (i),we get:x=5.

Rate in still water =1/2(11+5)kmph=8kmph.

Rate of current=1/2(11-5)kmph=3kmph

20. ALLIGATION OR MIXTURE

IMPORTANT FACTS AND FORMULAE

1. **Alligation**: It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.
2. **Mean Price**: The cost price of a unit quantity of the mixture is called the mean price.
3. **Rule of Alligation**: If two ingredients are mixed, then

$$\frac{\text{(Quantity of cheaper)}}{\text{(Quantity of dearer)}} = \frac{\text{(C.P. of dearer) - (Mean price)}}{\text{(Mean price) - (C.P. of cheaper)}}$$

We present as under:

C.P. of a unit quantity of cheaper   C.P. of a unit quantity of dearer

(c)   (d)

(m)

(d-m)   *(m-c)*

$\therefore$ (Cheaper quantity) : (Dearer quantity) = (d - m) : (m - *c*).

4. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations the quantity of pure liquid= $\left[\ x(1\text{-}y/x)\text{\^{}}n\right]$ units.

SOLVED EXAMPLES

Ex. 1. In what ratio must rice at Rs. 9.30 per kg be mixed with rice at Rs. 10.80 per kg so that the mixture be worth Rs. 10 per kg ?

Sol. By the rule of alligation, we have:

C.P. of 1 kg rice of 1st kind (in paise)   C.P. of 1 kg rice of 2nd kind (in paise)

930 . 1080 .

Mean pnce
(in paise)
1000

80 70

:.' Required ratio = 80 : 70 = 8 : 7.

Ex. 2. How much water must be added to 60 litres of milk at 1 ½ litres for Rs. 2 So as to have a mixture worth Rs.10 2/3 a litre ?

Sol. C.P. of 1 litre of milk = Rs. (20 x 2/3) = Rs. 40/3

c.p of 1 litre of milk c.p of 1 litre of milk

0 Rs. $\frac{40}{3}$

Mean price
(Rs. $\frac{32}{3}$)

(40/3-32/3)=8/3 (32/3-0)=32/3

$\therefore$ Ratio of water and milk = $\frac{8}{3}$: $\frac{32}{3}$ = 8 : 32 = 1 : 4

$\therefore$ Quantity of water to be added to 60 litres of milk = $\left[1/4 \text{ X } 60\right]$ litres =15 litre

Ex. 3. In what ratio must water be mixed with milk to gain 20 % by selling the mixture at cost price?

Sol. Let C.P. of milk be Re. 1 per litre.

Then, S.P. of 1 litre of mixture = Re. 1.

Gain obtained = 20%.

$\therefore$ C.P. of 1 litre of mixture = Rs. $\left[(100/120)^* \ 1\right]$ =Rs.5/6

By the rule of alligation, we have:

C.P. of 1 litre of water   C.P. of1litre ofmilk

0   Re.1

(Re. 5/6)

(1- (5/6))= 1/6   ((5/6)-0)=5/6

∴Ratio of water and milk = 1/6 : 5/6 =

Ex. 4. .How many kgs. of wheat costing Rs. 8 per kg must be mixed with 86 kg of rice costing Rs. 6.40 per kg so that 20% gain may be obtained by Belling the mixture at Rs. 7.20 per kg ?

Sol. S.P. of 1 kg mixture = Rs. 7.20,Gain = 20%.

∴ C.P. of 1 kg mixture = Rs.[(100/120)*7.20]=Rs. 6.

By the rule of alligation, we have:

C_P. of 1 kg wheat of 1st kind   C.P. of 1 kg wheat of 2nd kind

(800p) .   (540 p)

Mean price

(600 p)

60   200

Wheat of 1st kind: Wheat of 2nd kind = 60 : 200 = 3 : 10.

Let x kg of wheat of 1st kind be mixed with 36 kg of wheat of 2nd kind.

Then, 3 : 10 = x : 36 or lOx = 3 * 36 or x = 10.8 kg.

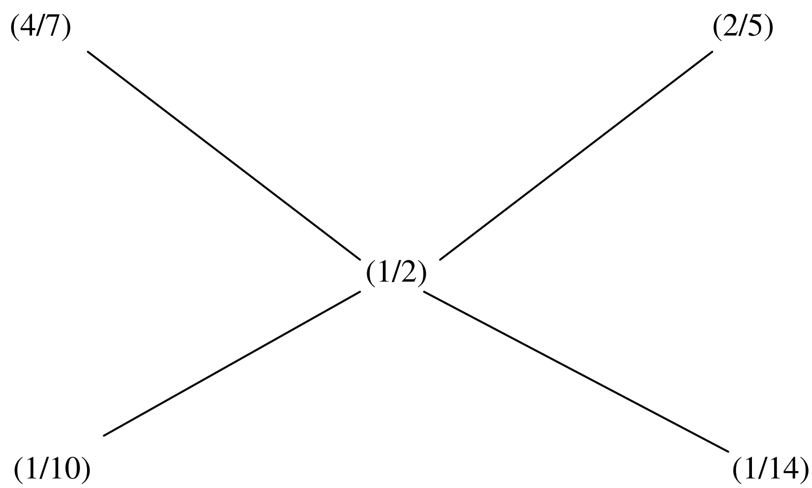
Ex. 5. The milk and water in two vessels A and B are in the ratio 4 : 3 and 2: 3 respectively. In

what ratio, the liquids in both the vessels be mixed to obtain a new mixture in vessel C containing half milk and half water?

Sol. Let the C.P. of milk be Re. 1 per litre

Milk in 1 litre mixture of A = 4/7litre; Milk in 1 litre mixture of B = 2/5 litre;
Milk in 1 litre mixture of C = ½ litre
C.P. of 1 litre mixture in A = Re .4/7*;* C.P. of 1 litre mixture in B = Re.2/5
Mean price = Re.1/2
By the rule of alligation, we have:

C.P. of 1 litre mix. in A C.P. of 1 litre mix. in B



Required ratio = 1/10 : 1/14 = 7 : 5